



# Simplified Collector Performance Model

Prediction of the thermal output of various solar collectors:

The quantity of thermal energy produced by any solar collector can be described by the energy balance equation

$$\dot{Q}_{out} = \dot{Q}_{opt} - \dot{Q}_{loss}$$

where

$\dot{Q}_{out}$  = rate of thermal energy output

$\dot{Q}_{opt}$  = rate of optical energy absorbed by receiver

$\dot{Q}_{loss}$  = rate of thermal energy lost from receiver.



# Optical Energy Absorbed by the Receiver

$$Q_{opt} = A_a \rho_{s,m} \tau_g \alpha_r R S I_a$$

$A_a$  = specular reflectance of concentrating mirror, if any (1.0 for non-concentrating flat-plate collector)

$\rho_{s,m}$  = transmittance of any glass envelope covering the receiver (e.g. glass cover plate in a flat-plate collector)

$A_a$  = aperture area of the collector

$S$  = receiver shading factor (fraction of collector aperture not shadowed by the receiver; 1.0 for a flat-plate collector)

$R$  = receiver intercept factor (fraction of reflected beam intercepted by receiver; 1.0 for a flat-plate collector)

$I_a$  = isolation (irradiance) incident on collector aperture (W/m<sup>2</sup>)

$\alpha_r$  = absorbance of the receiver

$S$ ,  $R$ ,  $\alpha_r$ ,  $\rho_{s,m}$  and  $\tau_g$  are constants (in a more detailed model, the dependence of the angle of the incident insolation can be considered) dependent only on the materials used and the structure accuracy of the collector - they can be lumped into a single constant term  $\eta_{opt}$ , the optical efficiency of the collector. For a flat-plate collector utilizing no reflectors,  $S$ ,  $R$  and  $\rho_{s,m}$  are set equal to 1.0.



# Thermal Energy Lost from the Receiver

$$\dot{Q}_{loss} = A_r U_l (T_r - T_a)$$

$A_r$  = surface area of the receiver (m<sup>2</sup>)

$T_r$  = averaged receiver temperature (°C)

$T_a$  = ambient temperature (°C)

$U_l$  = overall heat loss coefficient (W/m<sup>2</sup> °C)

$$T_r = \frac{T_{out} + T_{in}}{2}$$

$T_{out}$  is the temperature in degrees C of the fluid leaving the collector while  $T_{in}$  is the temperature of the fluid entering the collector.

The heat loss coefficient  $U_l$  is not a simple constant but instead may vary as heat-loss mechanisms change with temperature. For example as the temperature increases, radiant heat loss from the receiver increases.



# Collector Efficiency

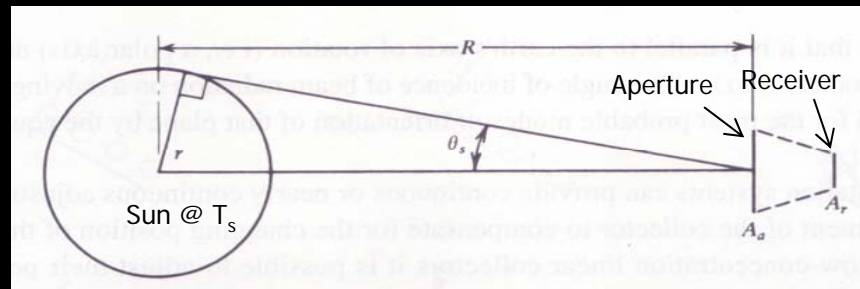
$$\dot{Q}_{out} = A_a \eta_{opt} I_a - A_r U_l (T_r - T_a)$$

$$\eta_{col} = \frac{\dot{Q}_{out}}{A_a I_a} = \eta_{opt} - \frac{A_r U_l (T_r - T_a)}{A_a I_a}$$

$$\eta_{col} = \eta_{opt} - \frac{U_l (T_r - T_a)}{C_g I_a}$$

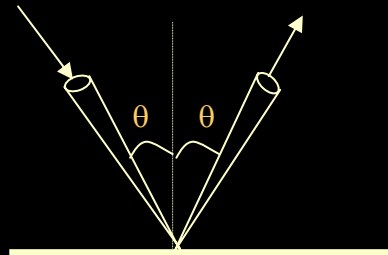
Where  $\eta_{col}$  is the aperture efficiency of the collector and  $A_r/A_a$  is the geometric concentration ratio ( $C_g$ )

$$C_g = \frac{A_a}{A_r}$$

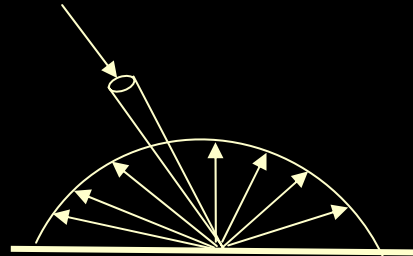




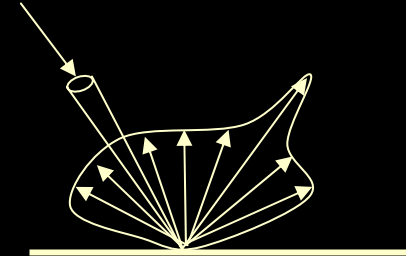
# Reflectance of Surfaces



Specular reflection



Diffuse reflection



General reflection

In general, the magnitude of the reflected intensity in a particular direction for a given surface is a function of the wave length and the spatial distribution of the incident radiation.

The transmittance, reflectance and absorption are functions of the incoming radiation, thickness, refractive index and extinction coefficient of the material. The refractive index  $n$ , and extinction coefficient of the material  $K$  of the cover material are functions of the wavelength of the radiation. However, for our purposes here, all properties will be assumed to be independent of the wavelength, which is the case for glass.



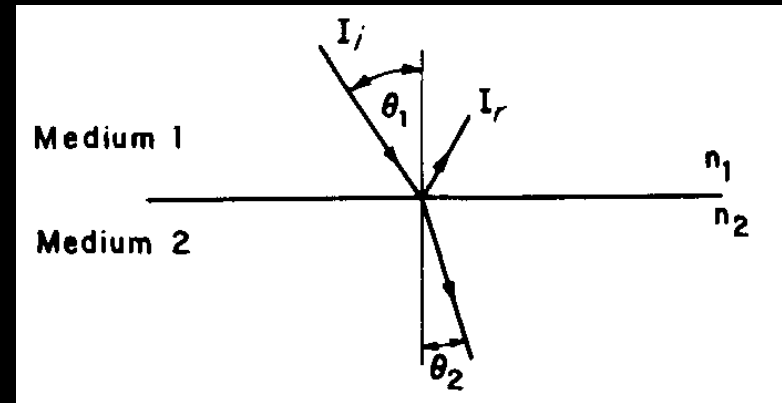
# Radiation Transmission and Absorption

Reflection of unpolarized Radiation:

$$r_{\text{perpendicular}} = r_1 = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)}$$

$$r_{\text{parallel}} = r_2 = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)}$$

$$r = \frac{I_r}{I_i} = \frac{1}{2}(r_1 + r_2)$$



For radiation at normal incidences:

$$r = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

$n$  is the refractive index of the medium

$$\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}$$

Snell's law

If one medium is air

$\theta_1$  and  $\theta_2$  are the angles of incidence and refraction

$$r = \frac{(n_1 - 1)^2}{(n_1 + 1)^2}$$

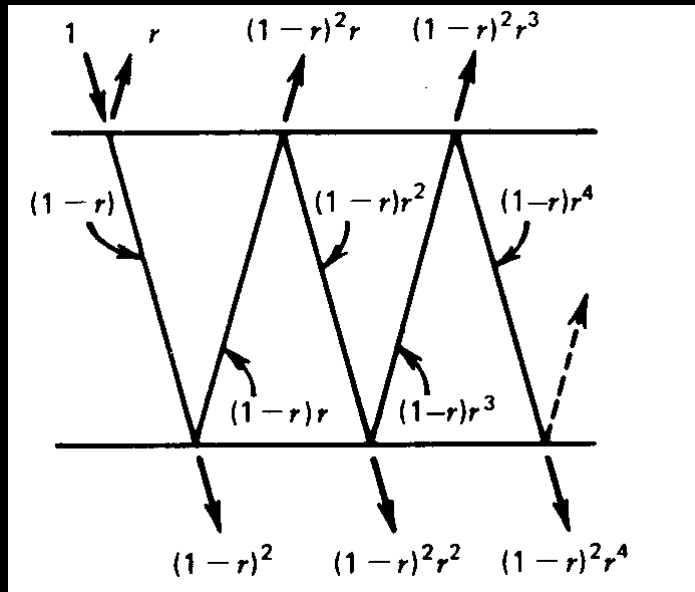




# Nonabsorbing Cover Transmission

Neglecting absorption in the cover material and considering only the perpendicular or parallel component of the polarization of the incoming radiation, the transmittance can be written as:

$$\tau_1 = \tau_2 = \frac{1 - r_1}{1 + r_1}$$





# Nonabsorbing Cover Transmission

Since the components of  $r_1$  and  $r_2$  are not equal (except at normal incidence), the transmittance of initially unpolarized radiation is the average transmission of two components,

$$\tau_r = \frac{1}{2} \left[ \frac{1-r_1}{1+r_1} + \frac{1-r_2}{1+r_2} \right]$$

where the subscript  $r$  is a reminder that only reflection losses are considered. For a system of  $N$  covers of the same material,

$$\tau_{rN} = \frac{1}{2} \left[ \frac{1-r_1}{1+(2N-1)r_1} + \frac{1-r_2}{1+(2N-1)r_2} \right]$$







## Example

The average index of refraction of glass for the solar spectrum is 1.526. Calculate the reflectance of one surface of glass at normal incidence and at 60 degrees. Also calculate the transmittance of two covers of nonabsorbing glass at normal incidence and at 60 degrees.

At normal incidence:

$$r = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} = \left(\frac{0.526}{2.526}\right)^2 = 0.0434$$

At an incidence angle of 60°

$$\theta_2 = \sin^{-1}\left(\frac{\sin 60}{1.526}\right) = 34.58$$

$$r = \frac{1}{2}(r_1 + r_2) = \frac{1}{2}\left[\frac{\sin^2(-25.42)}{\sin^2(94.58)} + \frac{\tan n^2(-25.42)}{\tan n^2(94.58)}\right] = 0.093$$

$$\tau_r(0) = \frac{1 - 0.0434}{1 + 3(0.0434)} = 0.85$$

$$\tau_r(60) = \frac{1}{2}\left[\frac{1 - 0.185}{1 + 3(0.185)} + \frac{1 - 0.001}{1 + 3(0.001)}\right] = 0.76$$



# Absorption by Glazing

The absorption of radiation in a partially transparent medium is described by Bouguer's law, which is based on the assumption that the absorbed radiation is proportional to the local intensity in the medium and the distance  $x$  the radiation has traveled in the medium:

$$dI = -IKdx$$

Where  $K$  is the extinction coefficient ( $4m^{-1} \sim 32m^{-1}$ ) which is assumed to be constant in the solar spectrum. Integrating along the actual path length in the medium from  $0$  to  $L/\cos \theta_2$  yields:

$$\tau_a = \frac{I_{transmitted}}{I_{incident}} = e^{-\left(\frac{KL}{\cos \theta_2}\right)}$$





# Optical Properties of Cover Systems

$$\tau_{perpendicular} = \tau_a \frac{1 - r_{perpendicular}}{1 + r_{perpendicular}} \left( \frac{1 - r_{perpendicular}^2}{1 - (r_{perpendicular} \tau_a)^2} \right)$$

Transmittance

$$\rho_{perpendicular} = r_{perpendicular} (1 + \tau_a \tau_{perpendicular})$$

Reflectance

$$\alpha_{perpendicular} = (1 - \tau_a) \left( \frac{1 - r_{perpendicular}}{1 - r_{perpendicular} \tau_a} \right)$$

Absorptance

For practical collector covers:  $\tau_a > 0.9$  and  $r = 0.1$

The transmittance of a single cover  $\approx \tau_a \tau_r$

The absorptance of the collector cover,  $\alpha \approx 1 - \tau_a$

The reflectance of a single cover,  $\rho \approx \tau_a - \tau$



## Example

Calculate the transmittance, reflectance and absorptance of a single glass cover 2.3 mm thick at an angle of 60 degrees. The extinction coefficient of the glass is  $32 \text{ m}^{-1}$ .

At an incidence angle of  $60^\circ$ , the optical path length is

$$\frac{KL}{\cos \theta_2} = 32 \times \frac{0.0023}{\cos 34.58} = 0.0894$$

Refraction angle calculated in an earlier example.

$$\tau_a = e^{-0.0894} = 0.915$$

The transmittance is found by averaging the transmittance of the parallel and perpendicular components of polarization or simply:

$$\tau = \frac{0.915}{2} \left[ \frac{1-0.185}{1+0.185} + \frac{1-0.001}{1+0.001} \right] = 0.771$$

The reflectance is found:

$$\alpha = 1 - 0.915 = 0.085$$

The absorptance is found to be:  $\rho = 1 - 0.771 - 0.085 = 0.144$

The above results are also valid to identical multiple covers. The length L equal to the total cover system thickness.





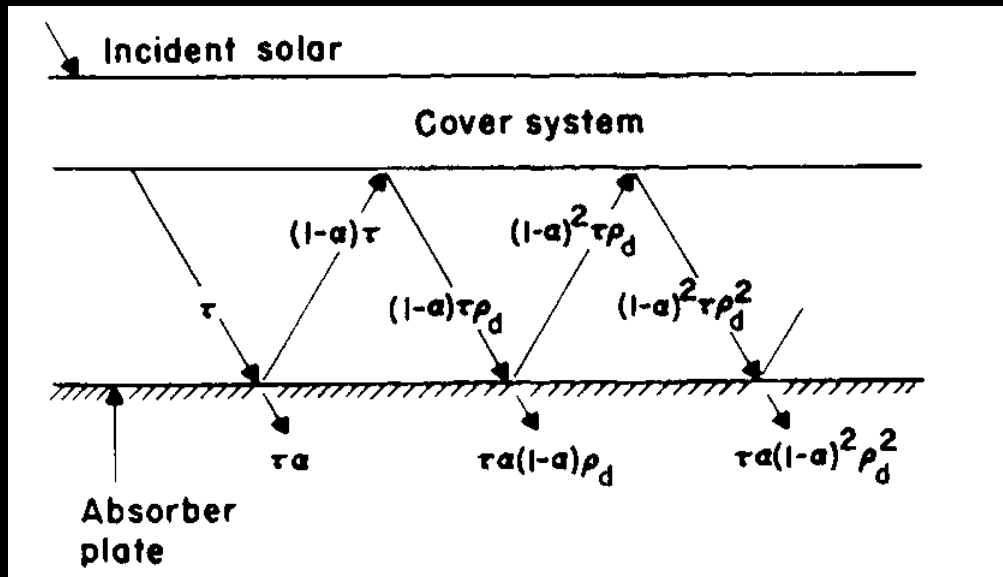
# Transmission - Absorption

Transmittance-absorption Product ( $\tau\alpha$ ):

Transmittance of the cover system at the desired angle =  $\tau$

The angular absorption of the absorber plate =  $\alpha$

Of the incident energy,  $\tau\alpha$  is absorbed by the absorber plate and  $(1-\alpha)\tau$  is reflected back to the cover system. The reflection from the absorber plate is diffusive.



$\rho_d$  refers to the reflectance of the cover system for diffuse radiation incident from the bottom side and can be estimated to be the difference between  $\tau_a$  and  $\tau$  at an angle  $60^\circ$ . The fraction of the incident energy ultimately absorbed is:

$$\tau\alpha = \frac{\tau\alpha}{1 - (1-\alpha)\rho_d}$$





# Transmittance-absorption Product

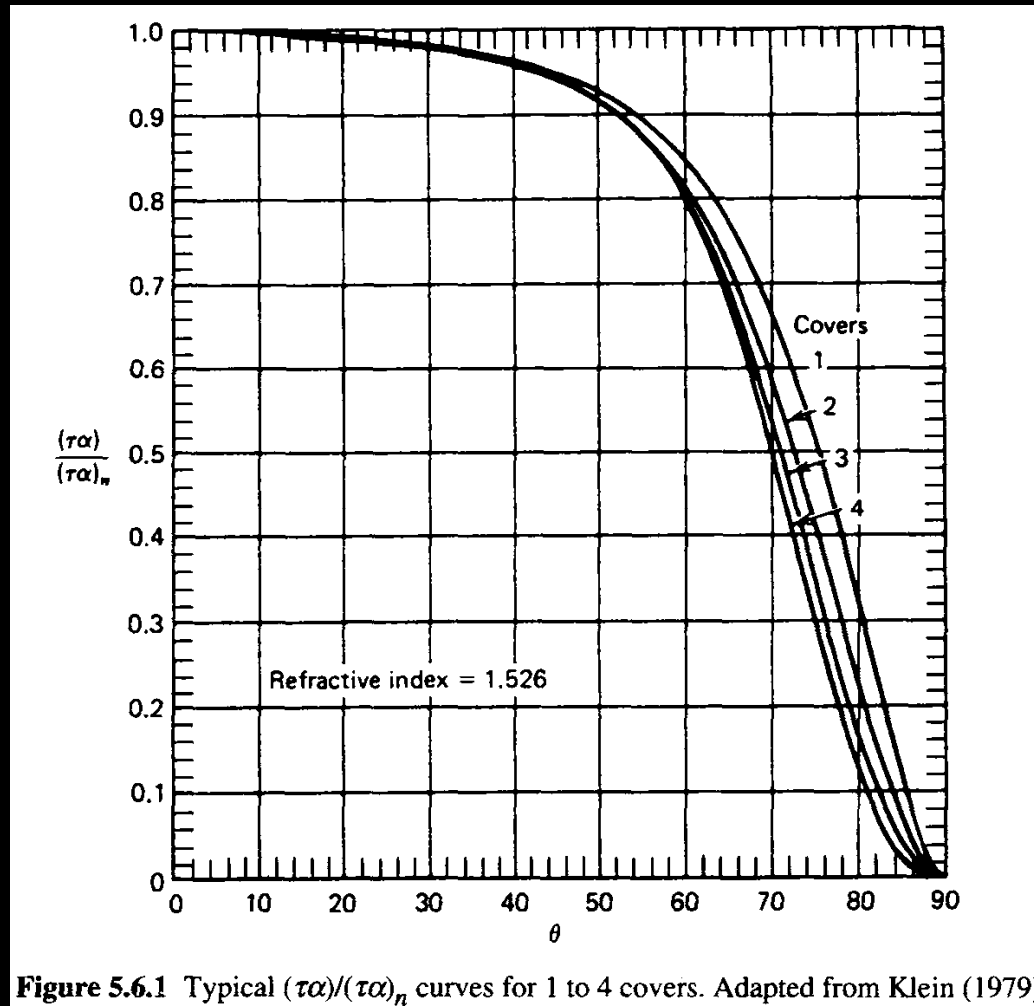
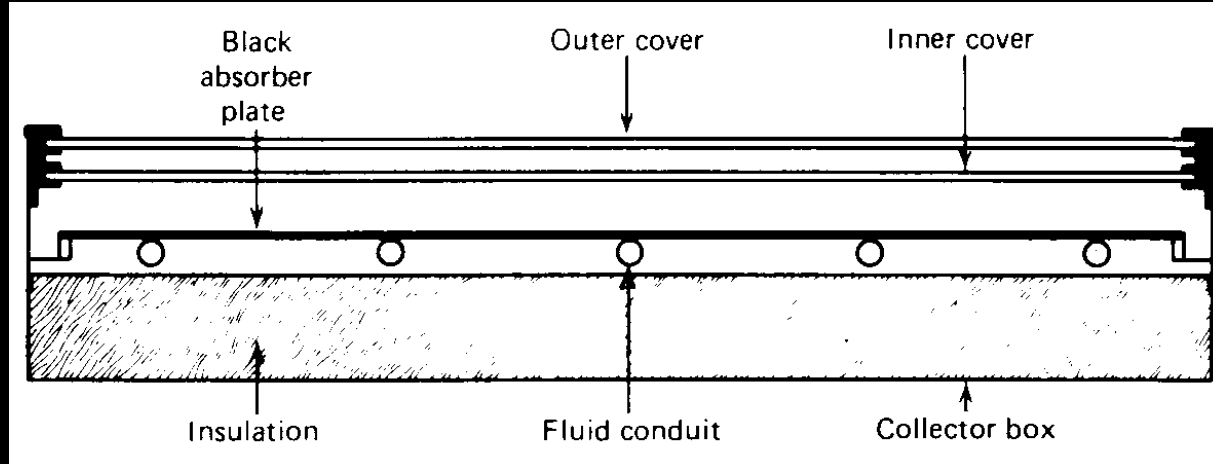


Figure 5.6.1 Typical  $(\tau\alpha)/(\tau\alpha)_n$  curves for 1 to 4 covers. Adapted from Klein (1979).



# Basic Flat-Plate Collector





# Flat-Plate Collectors

In the solar collector, the energy transfer is from source of radiant energy to a fluid.

The flux of incident radiation  $\sim 1000 \text{ W/m}^2$  ( it is variable)

The wavelength range:  $0.3 - 3.0 \mu\text{m}$

Energy delivery at moderate temperatures  $\sim 400\text{K}$

Use both beam and diffusive solar radiation - do not require tracking the Sun.

Major applications: solar water heating, building heating, air conditioning and industrial process heat.

Maximum temperature  $\sim 400\text{K}$



# Absorbed Radiation

The solar radiation absorbed by a collector per unit area of absorber  $S$  is equal to the difference between the incident solar radiation and the optical losses and is given by the expression:

$$S = I_b R_b (\tau\alpha)_b + I_d (\tau\alpha)_d \left( \frac{1 + \cos\beta}{2} \right) + \rho_g (I_b + I_d) (\tau\alpha)_g \left( \frac{1 - \cos\beta}{2} \right)$$

Where  $(1 + \cos\beta)/2$  and  $(1 - \cos\beta)/2$  are the view factors from the collector to the sky and from the collector to the ground respectively. The subscripts b, d, and g represent beam, diffuse and ground.

$\tau\alpha$  = transmittance -absorption product - a property of a cover-absorber combination.

The geometric factor  $R_b$  is the ratio of beam radiation on the tilted surface to that on a horizontal surface.

$I$  is the solar radiation  $J/m^2$  and  $\rho_g$  is the ground reflectance



# Energy Balance

The thermal energy lost from the collector to the surroundings by conduction, convection and infrared radiation =  $U_L(T_{pm} - T_a)$

Where,  $U_L$  is the heat transfer coefficient,  $T_{pm}$  and  $T_a$  are mean absorber plate temperature and ambient temperature respectively.

In steady state the useful energy out put of a collector of area  $A_c$  :

$$Q_u = A_c [S - U_L (T_{pm} - T_a)]$$

$T_{pm}$  is difficult to calculate or measure since it is a function of collector design, the incident solar radiation and the entering fluid conditions.

Collector performance:

$$\eta = \frac{\int Q_u dt}{A_c \int G_T dt}$$

← Incident total solar energy



# Energy Balance

**Collector heat removal factor** : Relates the **actual useful energy gain** of a collector to the useful gain if the whole collector surface were at the fluid inlet temperature.

$$F_R = \frac{\dot{m}C_p}{A_c U_L} \left[ 1 - \exp\left(-\frac{A_c U_L F'}{\dot{m}C_p}\right) \right]$$

$$Q_u = A_c F_R [S - U_L (T_i - T_a)]$$

$$F' = \frac{U_o}{U_L}$$

where  $U_o$  is the heat transfer resistance from the fluid to the ambient air.

**Collector flow factor:**

$$F'' = \frac{F_R}{F'}$$





# Collector Efficiency Factor

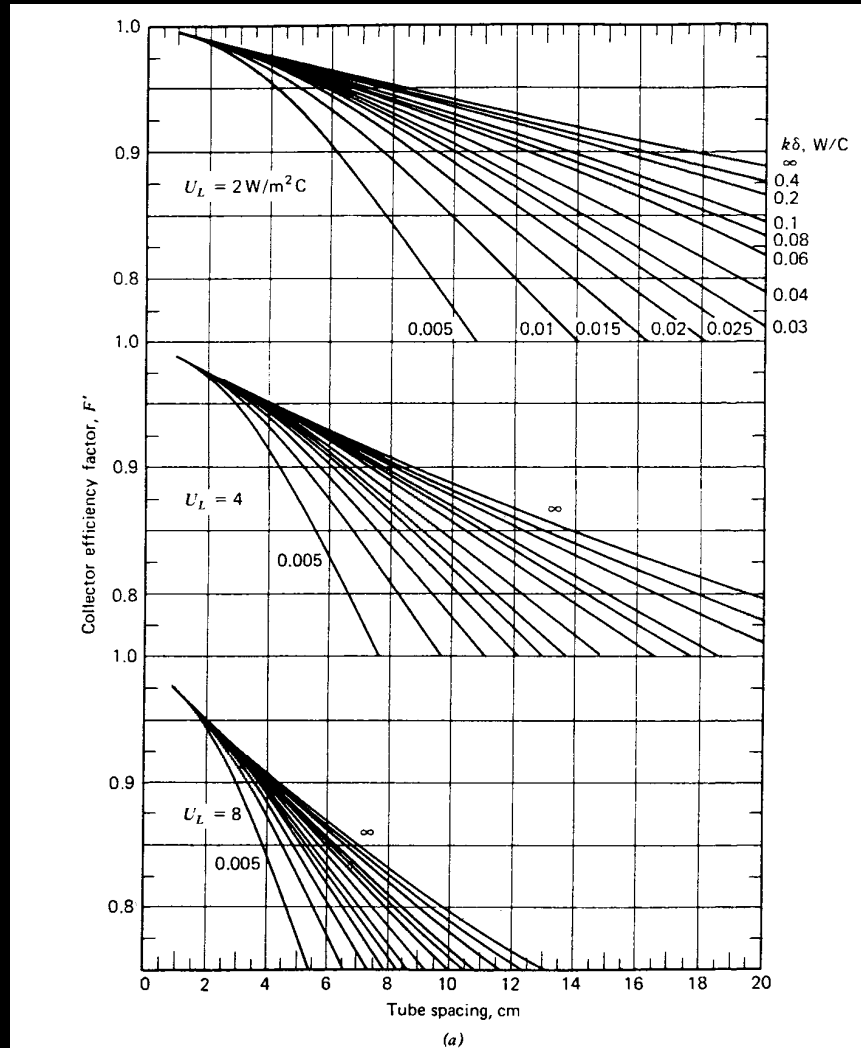
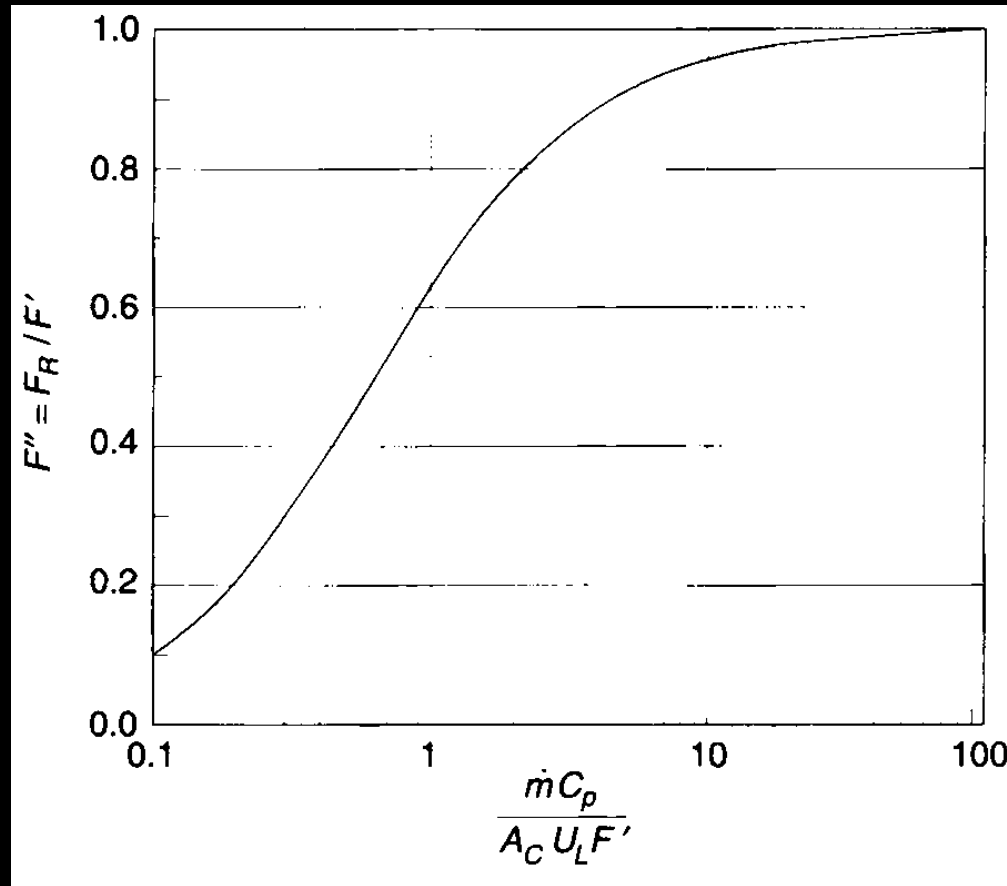


Figure 6.5.4 Collector efficiency factor  $F'$  versus tube spacing for 10 mm diameter tubes: (a)  $k_{\bar{r}} = 100 \text{ W/m}^2 \text{ C}$ ; (b)  $h_{fi} = 300 \text{ W/m}^2 \text{ C}$ ; (c)  $h_{fi} = 1000 \text{ W/m}^2 \text{ C}$



# Collector Flow Factor



# Energy Balance

$$Q_u = A_c F_R [S - U_L (T_i - T_a)]$$

Where  $T_i$  is the inlet fluid temperature and  $F_R$  is the heat removal factor.

Instantaneous efficiency:

$$\eta_i = \frac{Q_u}{A_c G_T} = \frac{F_R [G_T (\tau\alpha)_{av} - U_L (T_i - T_a)]}{G_T}$$

$G_T$  is given in  $W/m^2$





## Homework

Calculate the daily useful gain and efficiency of 10 solar collector modules installed “optimally” (first determine the optimum orientation) in parallel at the college of engineering. The hourly radiation on the plane of the collector  $I_T$ , the hourly radiation absorbed by the absorber plate  $S$ , and the hourly ambient temperature  $T_a$  are given in the table. For the collector assume the overall loss coefficient  $U_L$  to be  $8 \text{ W/m}^2\text{C}$  and the plate efficiency factor  $F'$  to be 0.841. The flow rate through each  $1 \times 2 \text{ m}$  collector panel is  $0.03 \text{ kg/s}$  and the inlet fluid temperature remains constant at  $40^\circ\text{C}$ .



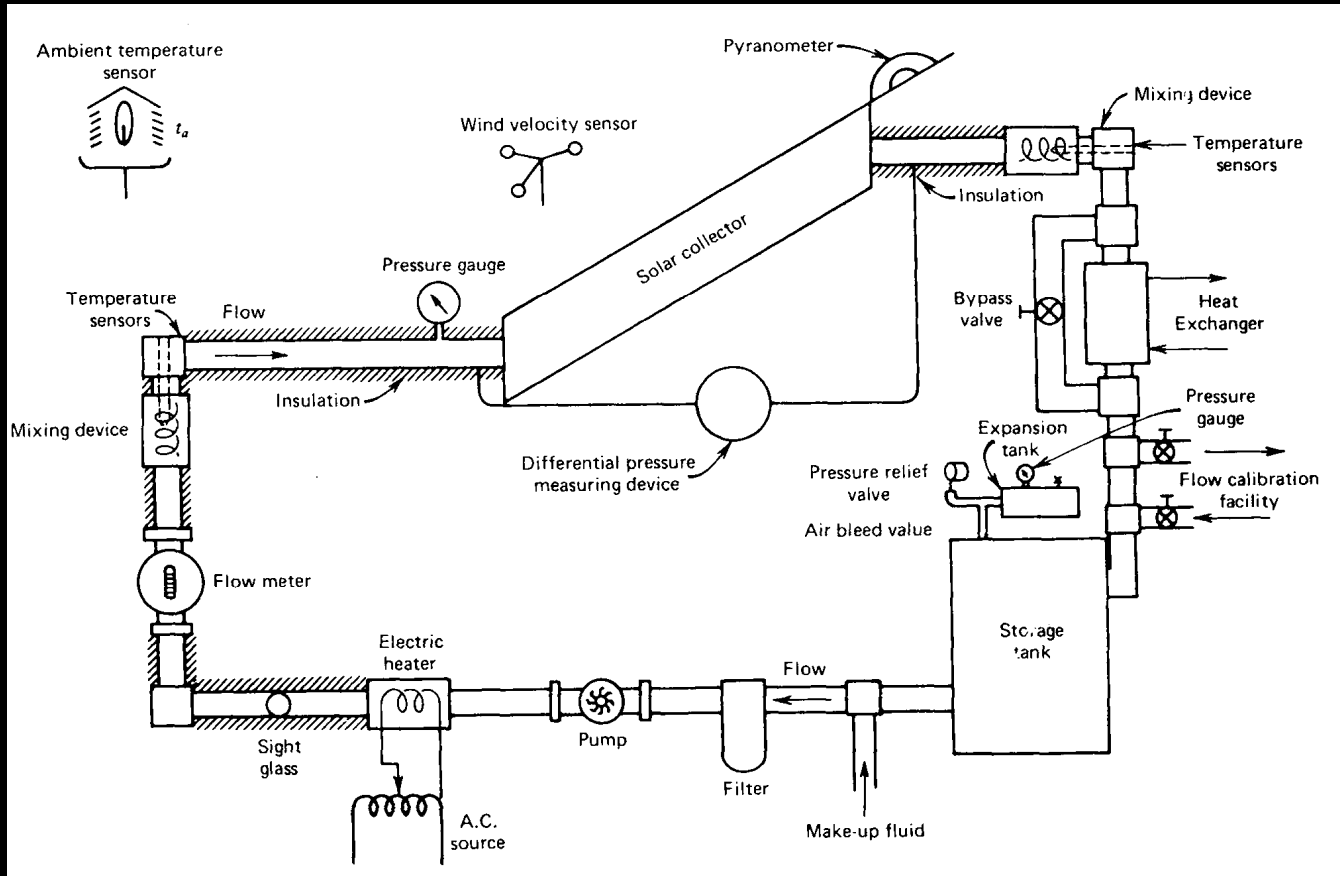
# Solar Collector Parameters

Time	$T_a$ C	$I_T$ MJ/m <sup>2</sup>	$S$ MJ/m <sup>2</sup>	$U_L(T_i - T_a)$ MJ/m <sup>2</sup>	$q_u$ MJ/m <sup>2</sup>	$\eta$
7-8	-11	0.02	-	-	0	0
8-9	-8	0.43	0.35	1.38	0	0
9-10	-2	0.99	0.82	1.21	0	0
10-11	2	3.92	3.29	1.09	1.76	0.45
11-12	3	3.36	2.84	1.07	1.42	0.42
12-1	6	4.01	3.39	0.98	1.93	0.48
1-2	7	3.84	3.21	0.95	1.81	0.47
2-3	8	1.96	1.63	0.92	0.57	0.29
3-4	9	1.21	0.99	0.89	0.08	0.07
4-5	7	<u>0.05</u>	-	0.95	<u>0</u>	-
Sum		19.79			7.57	





# Liquid Heating Collector System





# Collector Characterization

Determination of instantaneous  $h$  with beam radiation nearly normal to the absorbing surface.

Determination of effects of angle of incidence of the solar radiation.

Determination of collector time constant - a measure of effective heat capacity.

The useful gain:

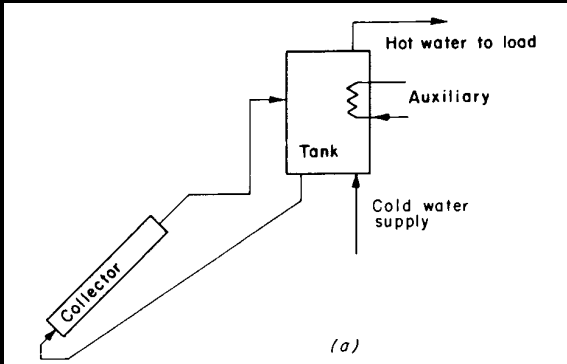
$$Q_u = m C_p (T_o - T_i)$$

Measure the fluid inlet and outlet temperatures and the fluid flow rate.

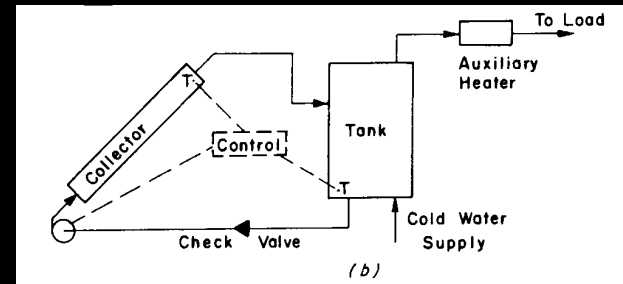




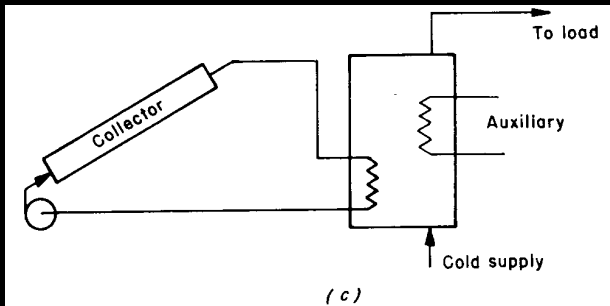
# Water Heater Configurations



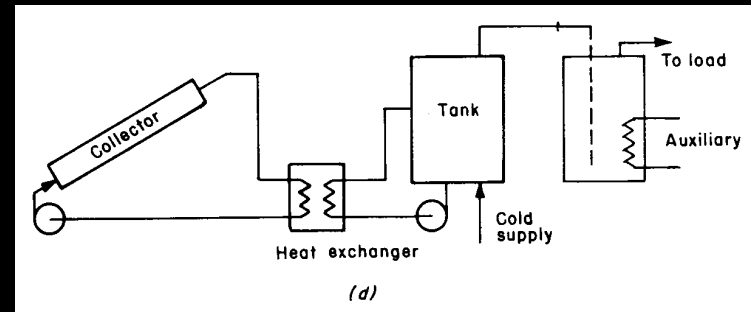
Natural circulation system



One tank forced circulation system



Antifreeze loop and internal heat exchanger



Antifreeze loop and external heat exchanger



# Design Problem Using TRNSYS

A transient process simulation program for the study of solar processes and its applications

Thermal performance of solar water heater

Develop a two cover collector design for a hot water load of 3000kg water/day at a minimum temperature of 335K evenly distributed between the hours of 0700-2100. Solar collector total effective area = 65 m<sup>2</sup>. The collector is designed to operate in Tallahassee.

