Solar Radiation

The objectives of this Lecture are:

1. to review the properties of solar radiation;
2. to determine theoretical upper limit of solar radiation available at the earth’s surface;
3. to determine the position of the sun in the sky and the beam radiation direction that is incident on surfaces of various orientations and shading.

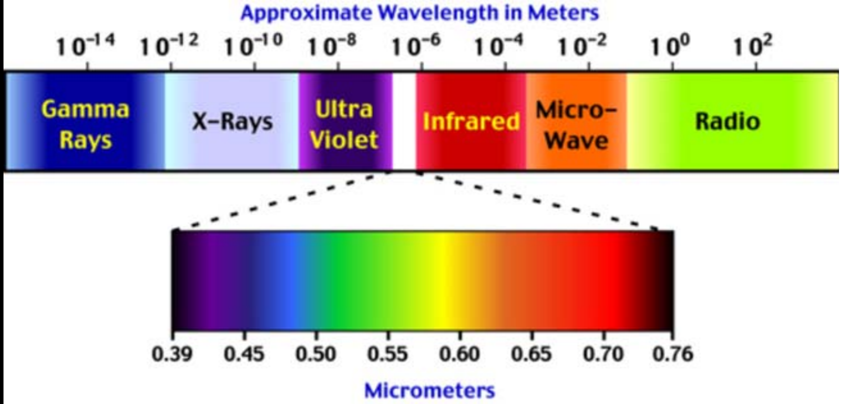
Source: Solar Engineering of Thermal Processes by Duffie & Beckman, John Wiley & Sons, 1991

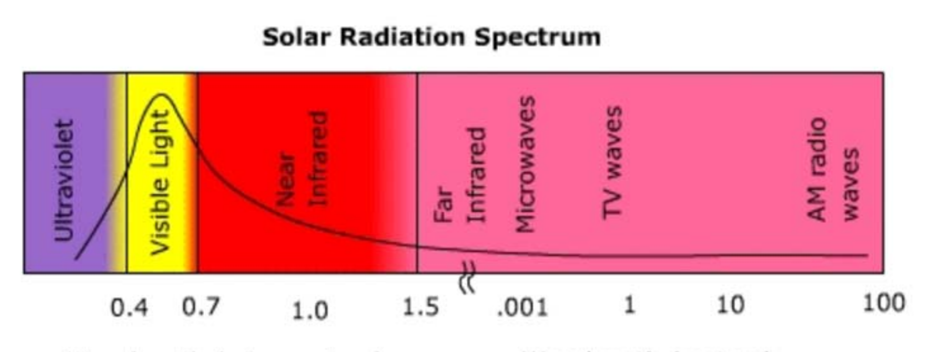
Reference: Principles of Solar Engineering, Goswami, Kreith and Kreider, Taylor & Francis, 2000

Energy and Radiation

Radiation: The transfer of energy via electromagnetic waves that travel at the speed of light. The velocity of light in a vacuum is approximately 3 x 10^8 m/s. The time it takes light from the sun to reach the Earth is 8 minutes and 20 seconds. Heat transfer by electromagnetic radiation can travel through empty space. Any body above the temperature of absolute zero (-273.15º C) radiate energy to their surrounding environment.

The many different types of radiation is defined by its wavelength. The electromagnetic radiation can vary widely.

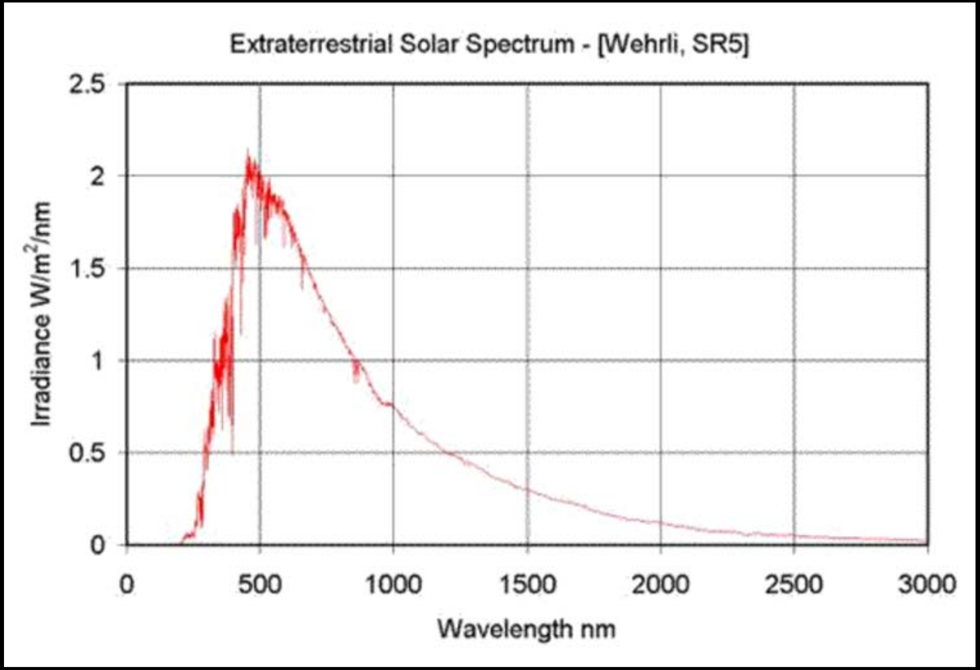




Source: http://www.uwsp.edu/geo/faculty/ritter/geog101/textbook/energy/nature_of_electromagnetic_radiation.html
http://www.physicalgeography.net

Sun Radiation Spectrum

Visible light has a wavelength of between 0.40 to 0.71 micrometers (µm). The sun emits only a portion (44%) of its radiation in this range. Solar radiation spans a spectrum from approximately 0.1 to 4.0 micrometers. About 7% of the sun's emission is in 0.1 to 0.4 micrometers wavelength band (UV). About 48% of the sun's radiation falls in the region between 0.71 to 4.0 micrometers (near infrared : 0.71 to 1.5 micrometers; far infrared: 1.5 to 4.0 micrometers).



Solar radiation incident outside the earth's atmosphere is called extraterrestrial radiation. On average the extraterrestrial irradiance is 1367 W/m^2. This value varies by ±3% as the earth orbits the sun.

Stephan-Boltzmann Law

The amount of electromagnetic radiation emitted by a body is directly related to its temperature. If the body is a perfect emitter (black body), the amount of radiation given off is proportional to the 4th power of its temperature as measured in degrees Kelvin. This natural phenomenon is described by the Stephan-Boltzmann law:

$$E = \sigma T^4$$

Where σ = 5.67 x 10^{-8} $Wm^{-2}k^{-4}$ and T is in K

In general, good emitters of radiation are also good absorbers of radiation at specific wavelength bands. This is especially true of greenhouse gases. Some objects in nature have almost completely perfect abilities to absorb and emit radiation. We call these objects black bodies. The radiation characteristics of the sun and the Earth are very close to being black bodies.

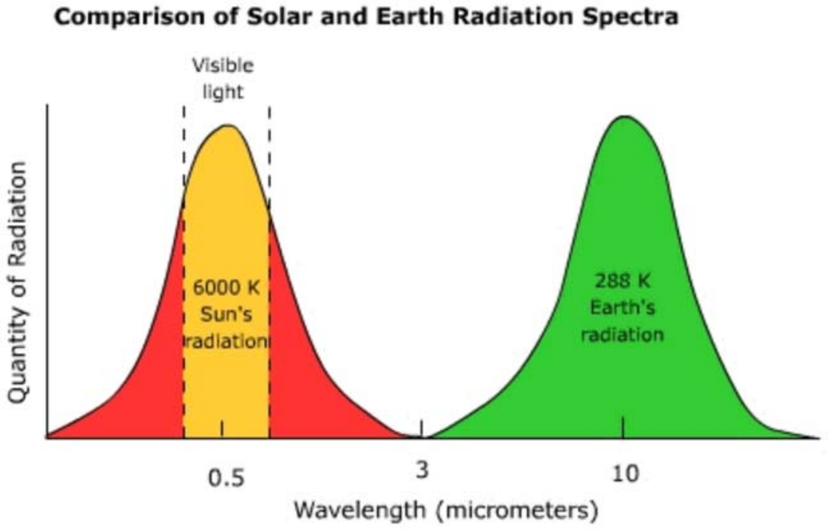
Wien’s Law

The wavelength of **maximum emission** of any body is inversely proportional to its absolute temperature. Thus, the higher the temperature, the shorter the wavelength of maximum emission. This phenomenon is often called Wien’s law:

$$\lambda_{\max} = \frac{C}{T}$$

$$C = 2897$$

where T is in Kelvin. According to the above equation the wavelength of maximum emission for the sun (5800 K) is about 0.5 μm, while the wavelength of maximum emission for the Earth (288 K) is approximately 10.0 μm.



The gases of the atmosphere are relatively good absorbers of long wave radiation and thus absorb the energy emitted by the Earth's surface. The absorbed radiation is emitted downward toward the surface as ***long wave atmospheric counter-radiation*** keeping near surfaces temperatures warmer than they would be without this blanket of gases. This is known as the ”greenhouse effect".

Inverse Square Law

The amount of radiation passing through a specific area is inversely proportional to the square of the distance of that area from the energy source. This phenomenon is called the inverse square law. Using this law we can model the effect that distance traveled has on the intensity of emitted radiation from a body like the sun.

$$Intensity = \frac{I}{d^2}$$

Where I is the intensity of radiation at one d *and* d *is the distance traveled*

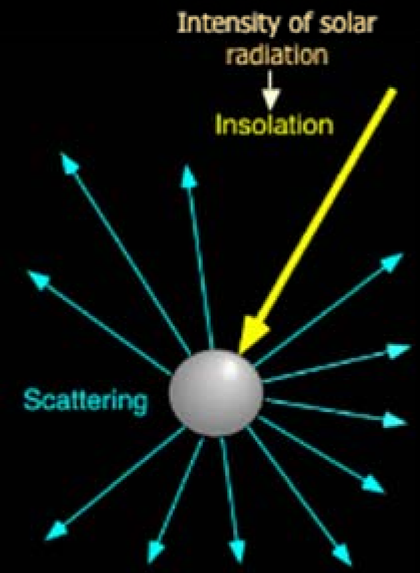
Radiation from the Sun is lessened by the inverse square law as it reaches further and further away from the Sun. So the further away that a planet is from the Sun then the less radiation it receives. What happens to that radiation depends on whether the planet has an atmosphere, whether the atmosphere contains clouds and how the clouds, or the surface, reflect the radiation.

For planets with no atmosphere all the Sun's radiation will strike the surface. Some of this will be reflected away from the planet but the rest will be absorbed. The temperature of the surface will be raised until there is equilibrium between the energy radiated by the warm surface of the planet and the received solar radiation. For planets like Mercury, this results in a very hot surface where the Sun is shining (more than 400°C) but very cold on the night side, where the radiation from the surface rapidly cools it to -180°C.

Atmospheric Effects on Incoming Solar Radiation

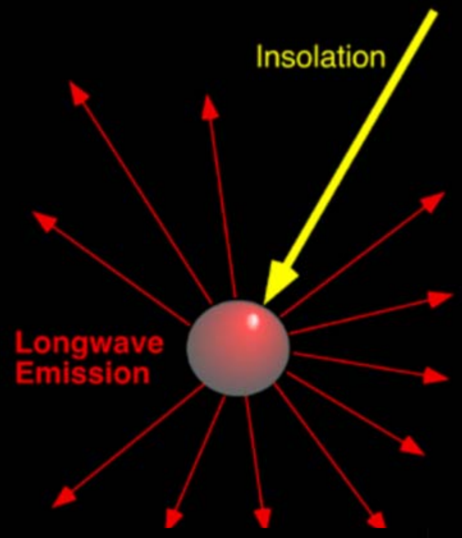
The Earth is a planet with an atmosphere and is largely transparent to the incoming solar radiation. There are constituents in the atmosphere which prevent some kinds of radiation from reaching the surface, such as ozone which stops the ultraviolet. A fair proportion of the Earth is covered by clouds which reflect a lot of the Sun's radiation and thus affecting the surface temperature.

The process of scattering occurs when small particles and gas molecules diffuse part of the incoming solar radiation in random directions without any alteration to the λ of the electromagnetic energy. Scattering does, however, reduce the amount of incoming radiation reaching the Earth's surface. A significant proportion of scattered shortwave solar radiation is redirected back to space. The amount of scattering that takes place is dependent on two factors: λ of the incoming radiation and the size of the scattering particle or gas molecule. In the Earth's atmosphere, the presence of a large number of particles with a size of about 0.5 μm results in shorter wavelengths being preferentially scattered. This factor also causes our sky to look blue because this color corresponds to those wavelengths that are best diffused. If scattering did not occur in our atmosphere the daylight sky would be black.

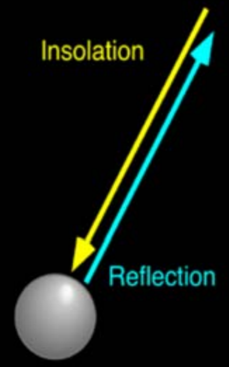


Atmospheric Effects on Incoming Solar Radiation

If intercepted, some gases and particles in the atmosphere have the ability to absorb incoming insolation. Absorption is defined as a process in which solar radiation is retained by a substance and converted into heat. The creation of heat also causes the substance to emit its own radiation. In general, the absorption of solar radiation by substances in the Earth's atmosphere results in temperatures that get no higher than 1800° C. Bodies with temperatures at this level or lower would emit their radiation in the longwave band. Further, this emission of radiation is in all directions so a sizable proportion of this energy is lost to space.



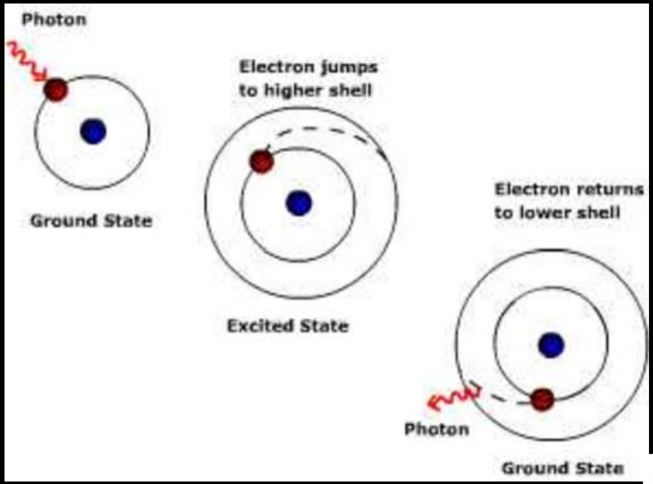
The third process in the atmosphere that modifies incoming solar radiation is reflection. Reflection is a process where sunlight is redirect by 180° after it strikes an atmospheric particle. This redirection causes a 100 % loss of the insolation. Most of the reflection in our atmosphere occurs in clouds when light is intercepted by particles of liquid and frozen water. The reflectivity (albedo) of a cloud can range from 40 to 90 %.



Selective Absorption of the Atmosphere

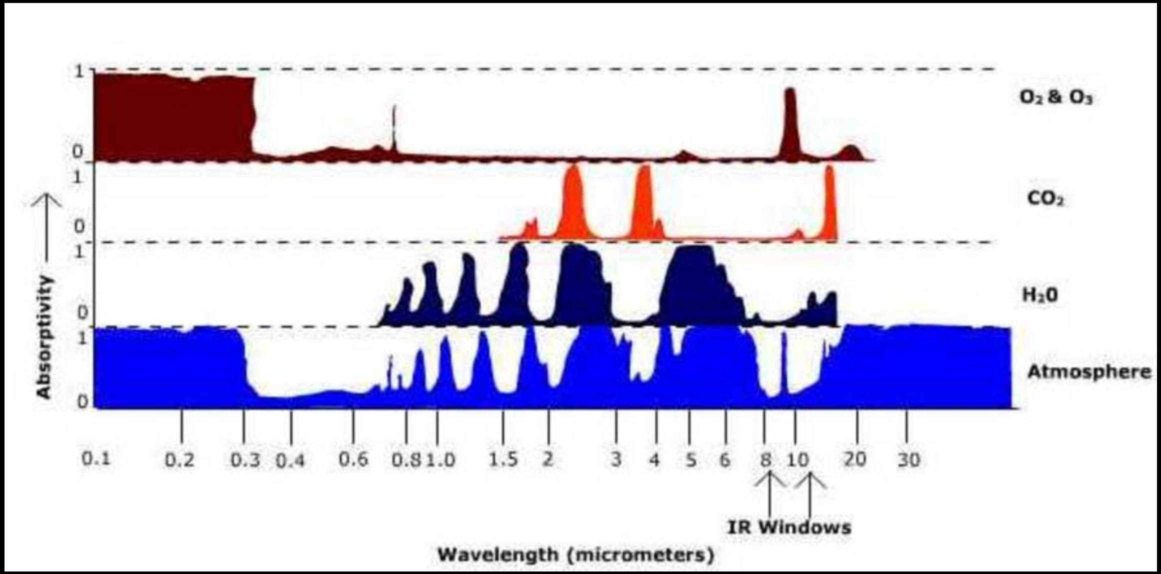
At the smallest scale the electromagnetic radiation behaves as a particle, like when light is emitted by a single atom or molecule. When energy is given off there is a change in the orbital pattern of the electrons that surround the nucleus of an atom. As the orbit changes, a bundle of energy called a "**photon**" is released. However, particles of light differ from particles of matter: they have no mass, occupy no space, and travel at the speed of light. The amount of energy carried by a photon varies inversely with wavelength, the shorter the wavelength the more energetic is the photon. Normally, light is formed from a large number of photons, with the intensity related to the number of them.

The gasses that comprise our atmosphere absorbs only particular wavelengths of light. Electrons orbit the nucleus of an atom at fixed orbital distances called orbital shells. The orbital shell for each atom is different and discrete. That is, for a given atom like hydrogen, its electrons can only orbit at particular distances and are different than those for atoms of other gases.



Selective Absorption of the Atmosphere

The amount of energy carried by a photon depends on the wavelength. Thus the atoms that comprise a gas can only absorb, or emit, particular wavelengths of energy (i.e. photons of energy). We can see this selective absorption by examining Figure below. The graph shows very little absorption for atmosphere as a whole in the shortwave end of the spectrum, especially in the visible light band (the band of maximum emission for the Sun). The atmosphere absorbs far better in the long wave end of the electromagnetic spectrum which is the region of maximum emission (10μm) for the Earth.

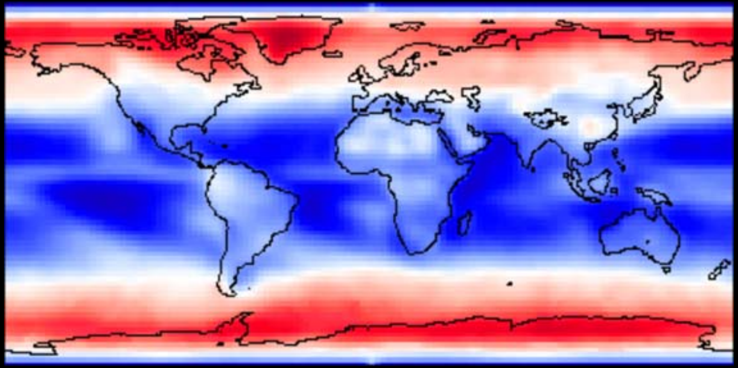


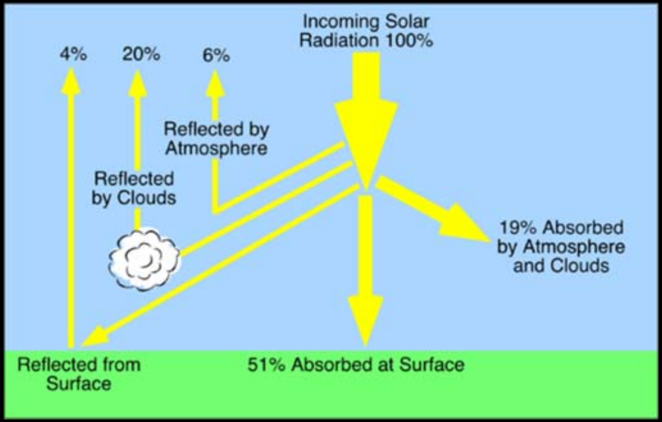
Atmospheric Effects on Incoming Solar Radiation

Sunlight reaching the Earth's surface unmodified by any of the atmospheric processes is termed direct solar radiation. Solar radiation that reaches the Earth's surface after it was altered by the process of scattering is called diffused solar radiation. Not all of the direct and diffused radiation is available at the Earth's surface. Some of the radiation received at the Earth's surface is redirected back to space by reflection.

Of all the sunlight that passes through the atmosphere annually, only 51 % is available at the Earth's surface; to heat the Earth's surface and lower atmosphere, evaporate water, and run photosynthesis in plants. Of the other 49 %, 4 % is reflected back to space by the Earth's surface, 26 % is scattered or reflected to space by clouds and atmospheric particles, and 19 % is absorbed by atmospheric gases, particles, and clouds.

The spatial pattern of surface reflectivity as measured for the year 1987.





Irradiance & Irradiation

Irradiance is given in *W/m*^{2} *and is represented by the symbol G.*

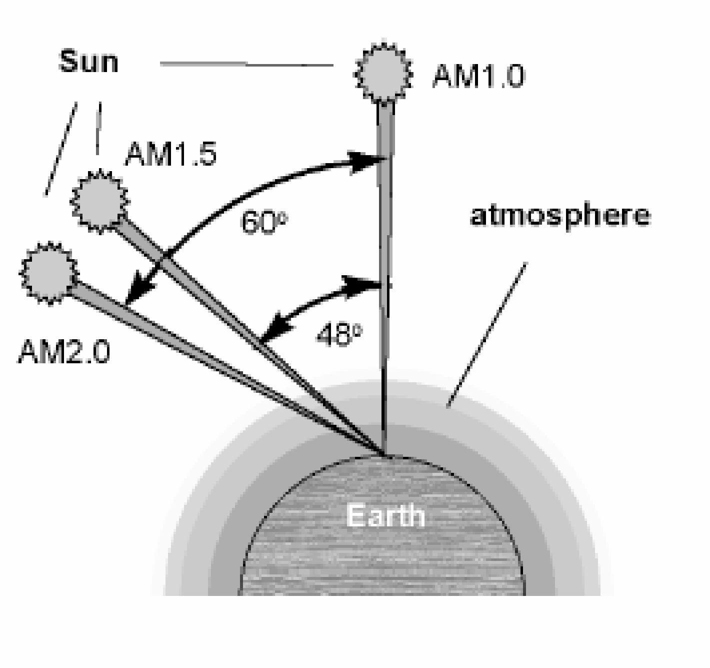
The rate at which radiant energy is incident on a surface per unit area of surface.

Irradiation is given in *J/m*^{2} and is the incident energy per unit area on a surface - determined by integration of irradiance over a specified time, usually an hour or a day.

Insolation is a term used to solar energy irradiation

Radiosity is the rate at which radiant energy leaves a surface, per unit area, by combined emission, reflection and transmission.

Air Mass



The path length of the solar radiation through the Earth's atmosphere in units of Air Mass (*AM*) increases with the angle from the zenith. The AM 1.5 spectrum is the preferred standard spectrum for solar cell efficiency measurements.

The easiest way to estimate the air mass in practice is to measure the length of the shadow *s* cast by a vertical structure of height *h* using

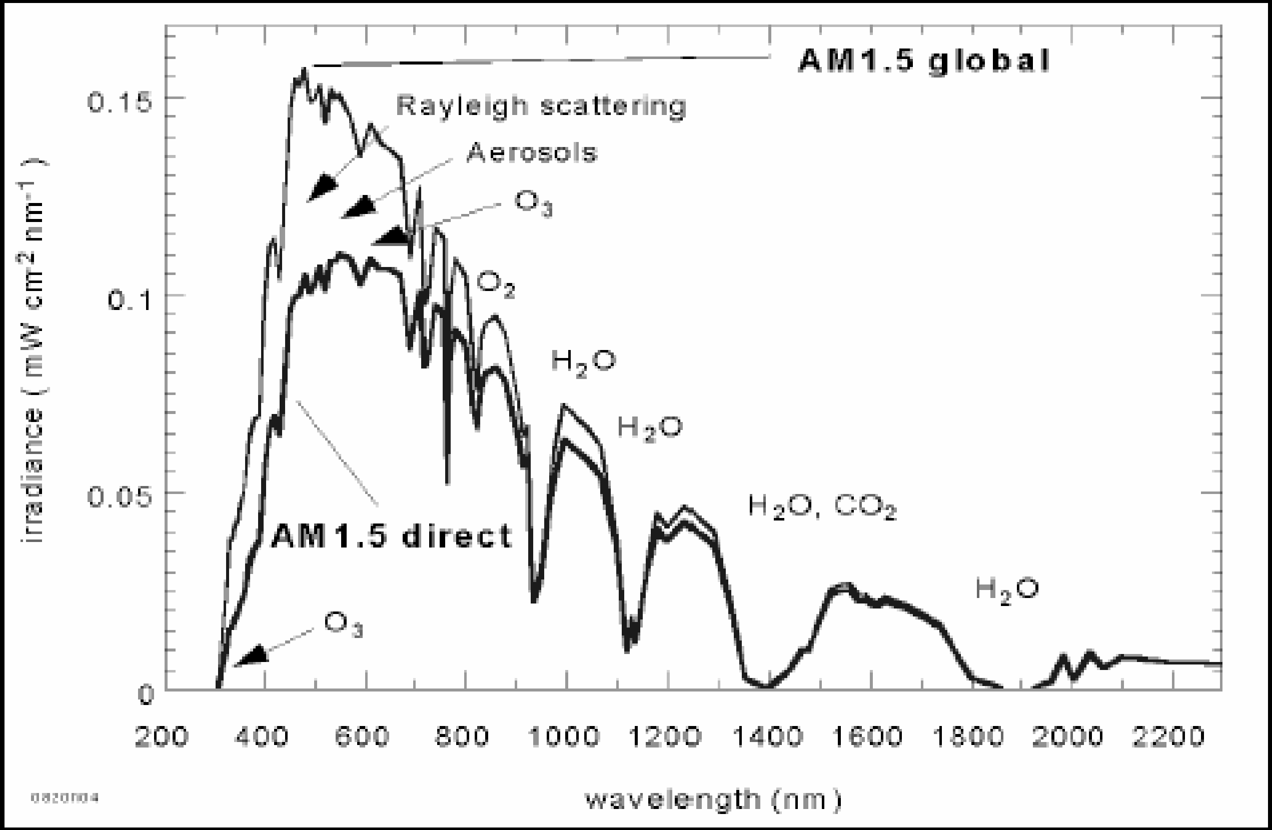
$$AM = \sqrt{1 + \left(\frac{s}{h}\right)^2}$$

Air Mass *AM* : The ratio of the mass of atmosphere through which beam radiation passes to the mass it would pass through if the sun were at zenith (directly overhead).

At sea level, AM =1 when the sun is at zenith; $AM = 2$ for a zenith angle θ_z of 60^o.

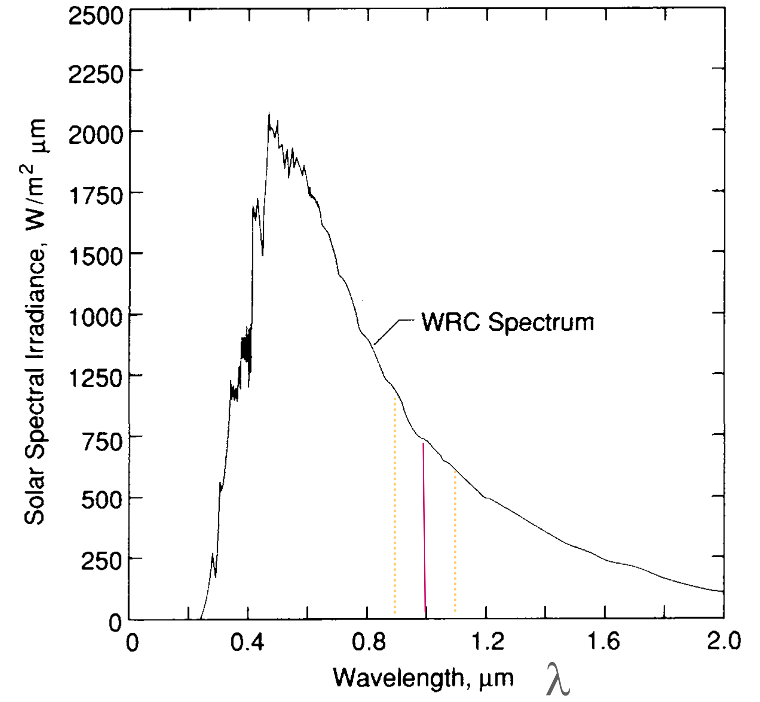
For $0 < \theta_z < 70^o$ AM= $1/\cos\theta_z$

Global Radiation



The global spectrum comprises the direct plus the diffused light.

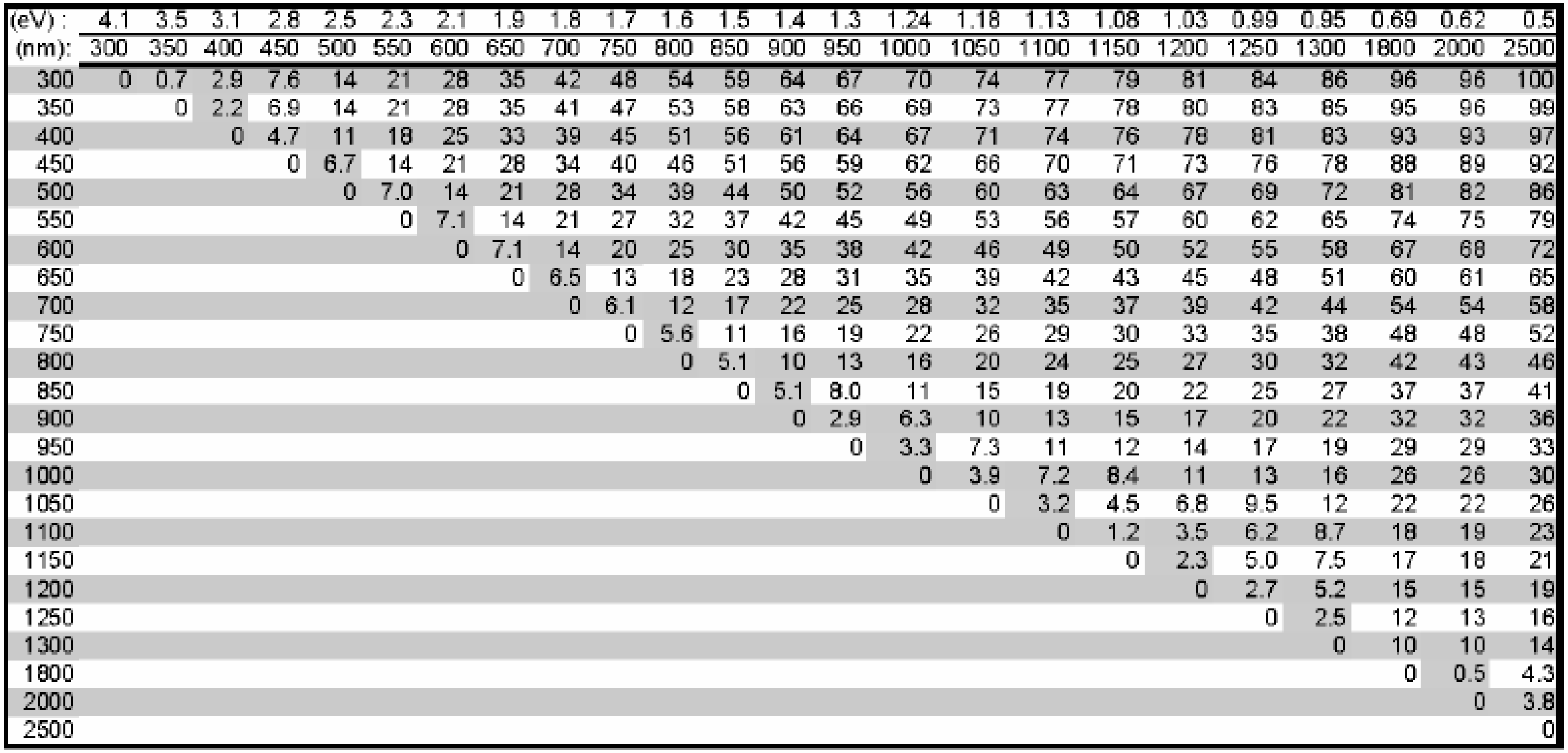
Spectral Distribution of Extraterrestrial Radiation*



$G_{SC,\lambda}$: The average energy over small bandwidths centered at wavelength λ; $F_{0\text{-}\lambda}$: The fraction of the total energy in the spectrum that is between wavelengths 0 and λ.

**The radiation that would be received in the absence of the atmosphere at mean-earth-sun distance* (World Radiation Center (WRC) standard)

AM 1.5d Spectrum Energy Distribution

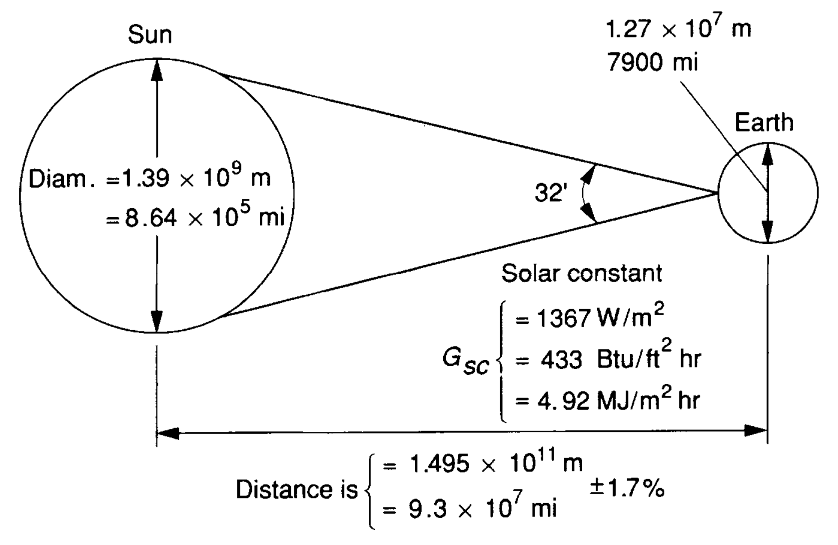


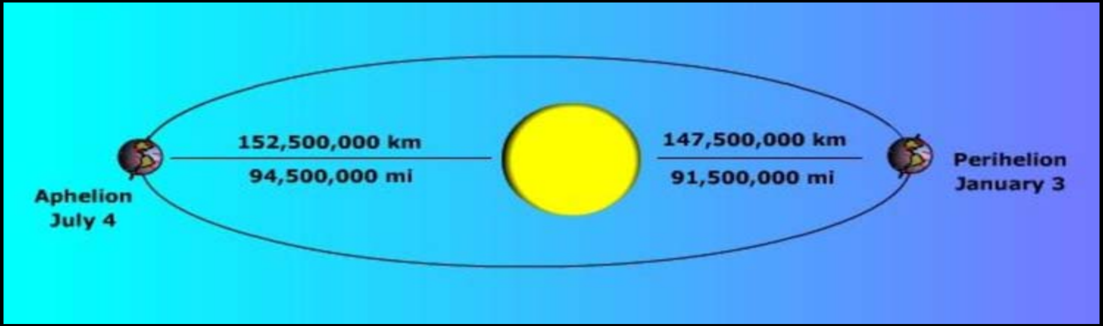
(eV) :	4.1	3.5	3.1	2.8	2.5	2.3	2.1	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.24	1.18	1.13	1.08	1.03	0.99	0.95	0.69	0.62	0.5
(nm):	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000	1050	1100	1150	1200	1250	1300	1800	2000	2500
300	0	0.7	2.9	7.6	14	21	28	35	42	48	54	59	64	67	70	74	77	79	81	84	86	96	96	100
350		0	2.2	6.9	14	21	28	35	41	47	53	58	63	66	69	73	77	78	80	83	85	95	96	99
400			0	4.7	11	18	25	33	39	45	51	56	61	64	67	71	74	76	78	81	83	93	93	97
450				0	6.7	14	21	28	34	40	46	51	56	59	62	66	70	71	73	76	78	88	89	92
500					0	7.0	14	21	28	34	39	44	50	52	56	60	63	64	67	69	72	81	82	86
550						0	7.1	14	21	27	32	37	42	45	49	53	56	57	60	62	65	74	75	79
600							0	7.1	14	20	25	30	35	38	42	46	49	50	52	55	58	67	68	72
650								0	6.5	13	18	23	28	31	35	39	42	43	45	48	51	60	61	65
700									0	6.1	12	17	22	25	28	32	35	37	39	42	44	54	54	58
750										0	5.6	11	16	19	22	26	29	30	33	35	38	48	48	52
800											0	5.1	10	13	16	20	24	25	27	30	32	42	43	46
850												0	5.1	8.0	11	15	19	20	22	25	27	37	37	41
900													0	2.9	6.3	10	13	15	17	20	22	32	32	36
950														0	3.3	7.3	11	12	14	17	19	29	29	33
1000															0	3.9	7.2	8.4	11	13	16	26	26	30
1050																0	3.2	4.5	6.8	9.5	12	22	22	26
1100																	0	1.2	3.5	6.2	8.7	18	19	23
1150																		0	2.3	5.0	7.5	17	18	21
1200																			0	2.7	5.2	15	15	19
1250																				0	2.5	12	13	16
1300																					0	10	10	14
1800																						0	0.5	4.3
2000																							0	3.8
2500																								0

Silicon solar cells with a bandgap of 1.13ev can maximally absorb 77% of the terrestrial solar energy.

Sun-Earth Relationships

The solar constant, G_{SC} is the energy from the sun, per unit time, received on a unit area of surface perpendicular to the direction of propagation of the radiation, at mean earth-sun distance, outside of the atmosphere.



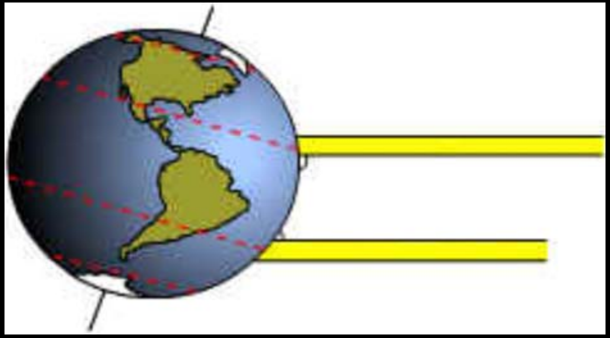


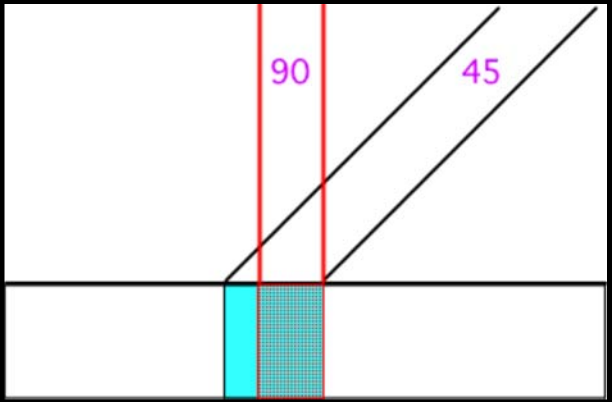
Earth's elliptic orbit

The elliptical path causes only small variations in the amount of solar radiation reaching the earth.

Sun-Earth Relationships

The Earth's axis is tilted 23 1/2 degrees from being perpendicular to the plane of the ecliptic. The axis of rotation remains pointing in the same direction as it revolves around the Sun, pointing toward the star Polaris. The constant tilt and parallelism causes changes in the angle that a beam of light makes with respect to a point on Earth during the year, called the "***sun angle***". The most intense incoming solar radiation occurs where the sun's rays strike the Earth at the highest angle. As the sun angle decreases, the beam of light is spread over a larger area and decreases in intensity. During the summer months the Earth is inclined toward the Sun yielding high sun angles. During the winter, the Earth is oriented away from the Sun creating low sun angles.

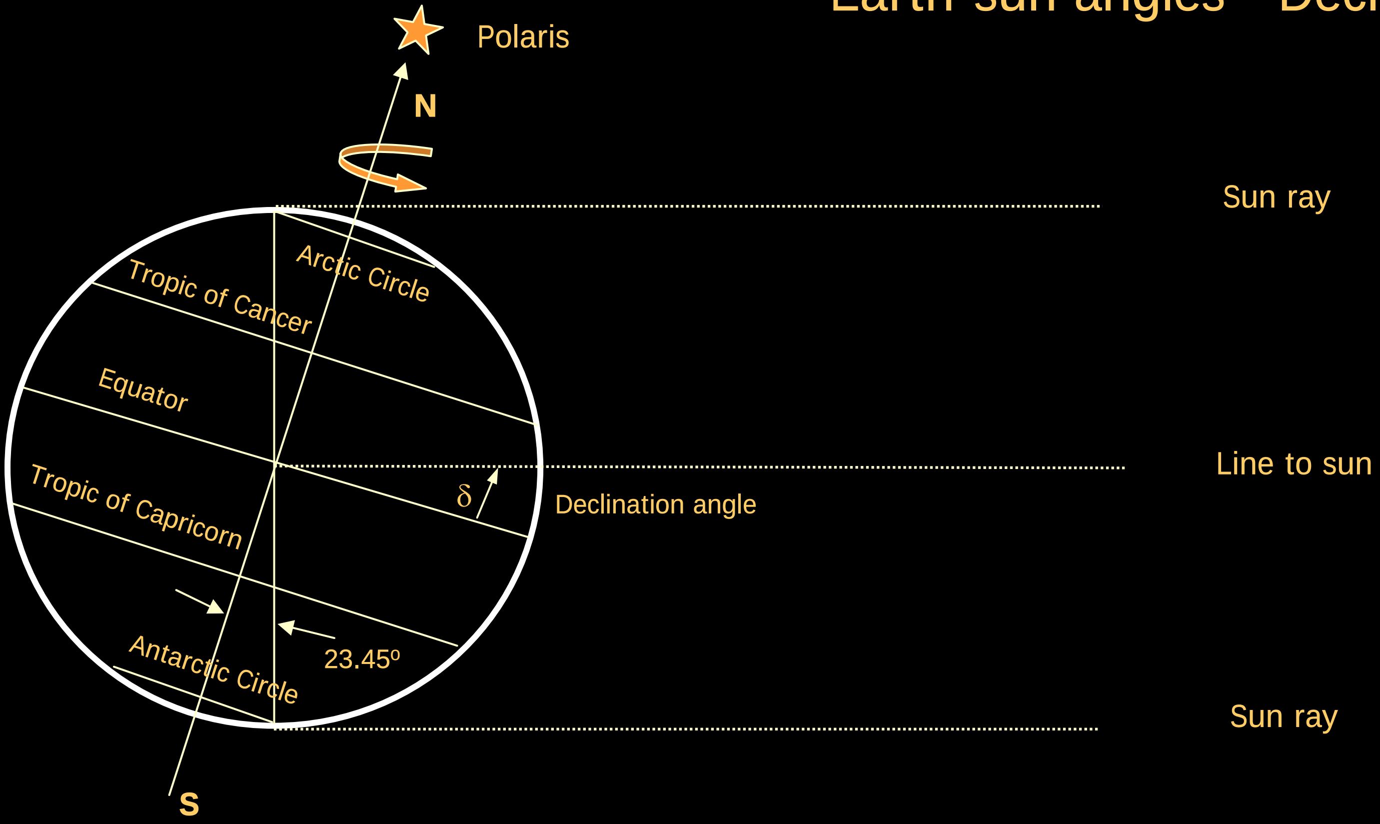




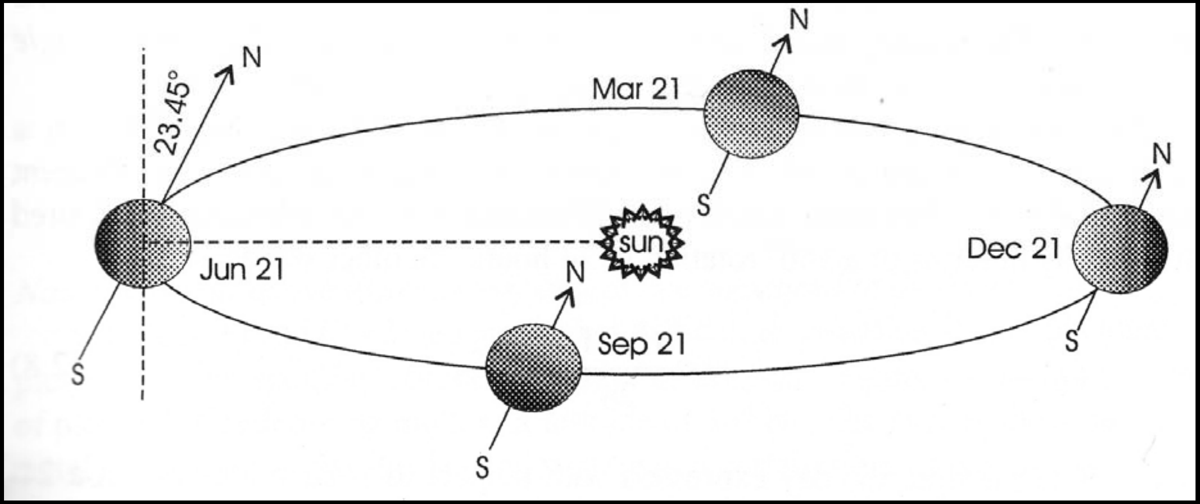
Sun angle determines the intensity of energy.

Sun-Earth Relationships

Earth-sun angles - Declination



Orbit of the Earth Around the Sun



At about June 21, we observe that the noontime time sun is at its highest point in the sky and the declination angle (δ) = +23.45°. We call this condition *summer solstice* and it marks the beginning of Summer in the northern hemisphere. The *winter solstice* occurs on about December 22 - the northern hemisphere is tilted away from the sun. The noontime sun at its lowest point in the sky, i.e the declination angle (δ) = -23.45°. In between, on about September 23 (autumnal equinox) and March 22 (vernal equinox) , the line from the earth to the sun lies on the equatorial plane and δ = 0°

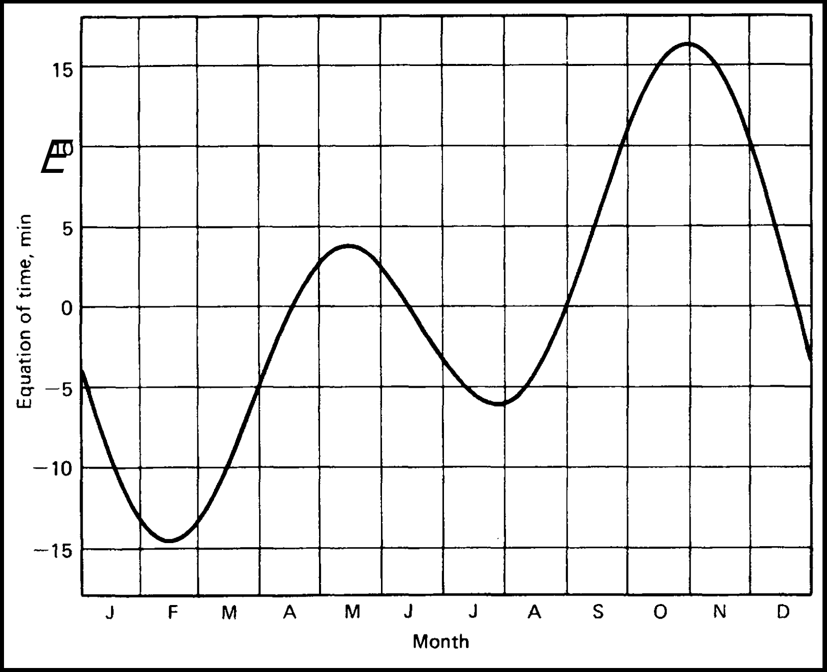
Source: Photovoltaic systems Engineering by Messenger & Ventre, CRC 2000.

Solar Time

Time based on the apparent angular motion of the sun across sky, with solar noon the time the sun crosses the meridian of the observer.

$$\text{Solar time - standard time} = 4(L_{st} - L_{loc}) + E$$

Where L_{st} is the standard meridian for local time zone. L_{loc} is the longitude of the location in question in degrees west. The time E is determined from the following figure.



Tallahassee, FL
Latitude: 30.438N
Longitude: 84.28W
Time zone: Eastern Daylight Saving

$$E = 229.2(0.000075 + 0.001868\cos B - 0.032077\sin B - 0.014615\cos 2B - 0.04089\sin 2B)$$

$$B = (n-1)\frac{360}{365}$$

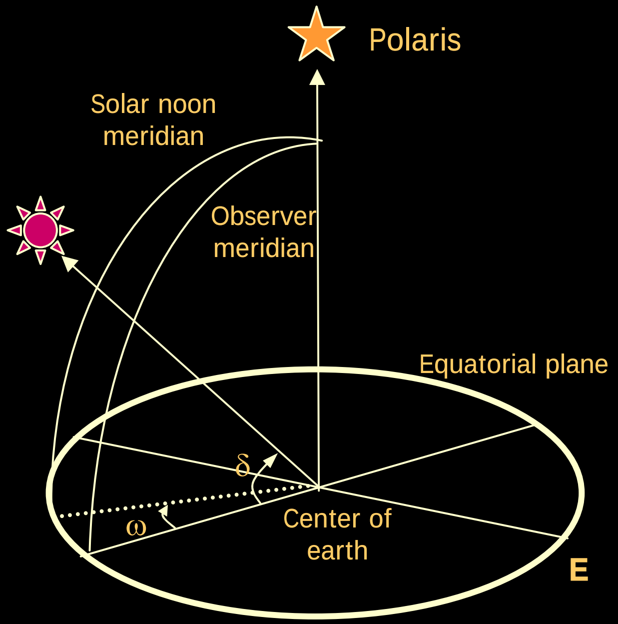
Where n is the day of the year

The Hour Angle (ω)

This is the angle defined as the angle between the plane of the meridian containing the point of interest and the meridian that touches the earth -sun line. The hour angle is zero at solar noon. The hour angle increases by 15 degrees every hour. An expression for the hour angle is

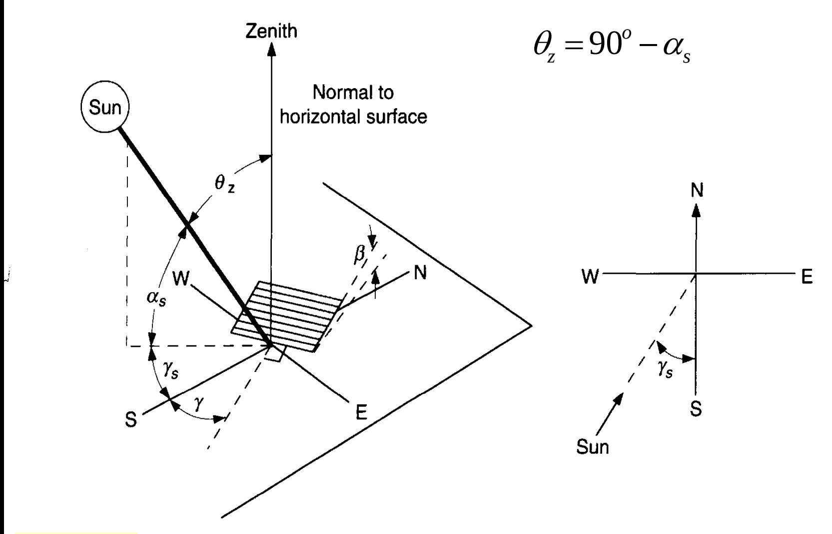
$$\omega = 15(t_s - 12) \quad \text{(degrees)}$$

Where t_s is the solar time in hours. For example, when it is 3 hours after solar noon, the hour angle has a value of 45 degrees. When it is 2 hours and 20 minutes before solar noon, the hour angle is 325 degrees.



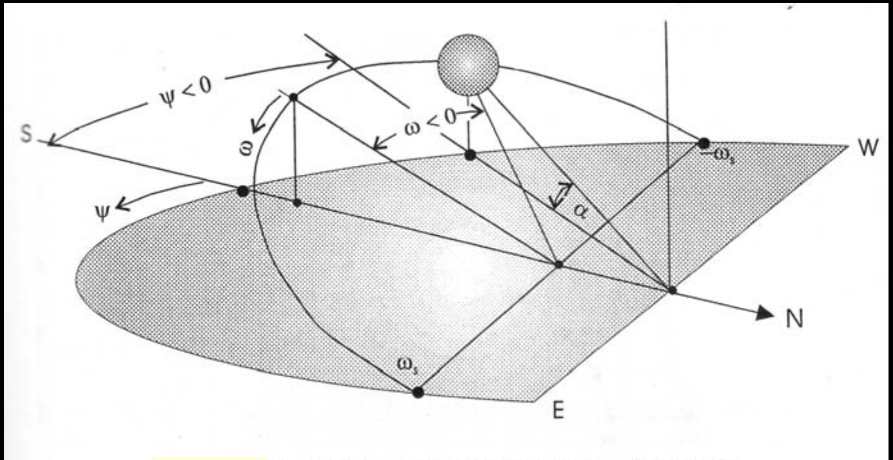
Observer-Sun Angles

Direction of Beam Radiation: The geometric relationships between a plane of any particular orientation relative to the earth at any time and the incoming beam solar radiation can be described in terms of several angles (Latitude(ϕ), Declination(δ), Slope(β), Surface azimuth angle(γ), Hour angle(ω), Angle of incidence(θ), Zenith angle(θ_z), Solar altitude angle (α_s) and solar azimuth angle (γ_s)).



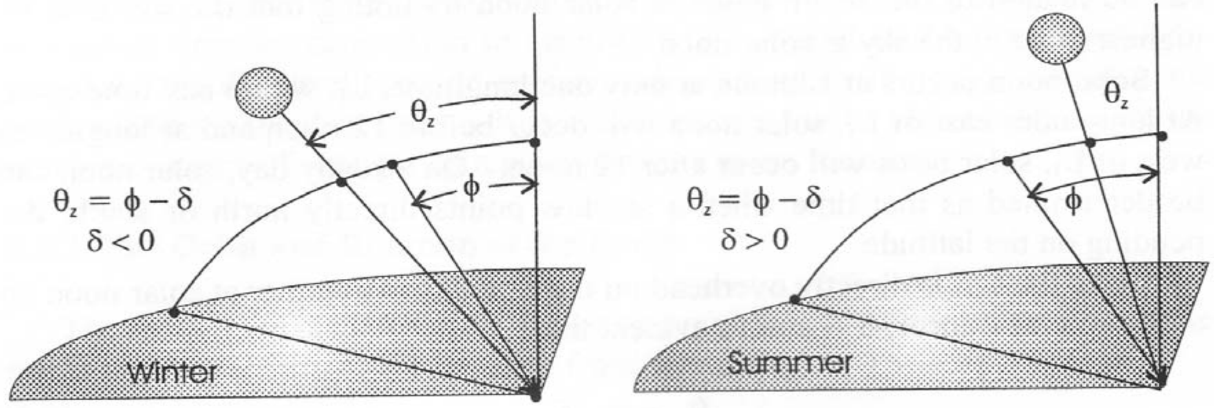
$$\theta_z = 90^o - \alpha_s$$

(a) Zenith angle, slope, surface azimuth angle, and solar azimuth angle for a tilted surface. (b) Plan view showing solar azimuth angle.



Sun angles, showing altitude, azimuth and hour angle.

Observer-Sun Angles

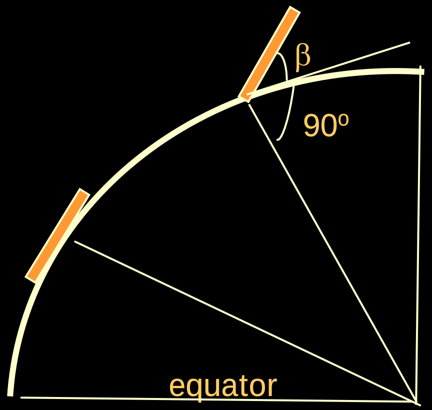


Relationships among θ_z, ϕ and δ at solar noon in winter and summer.

Source: Photovoltaic systems Engineering by Messenger & Ventre, CRC 2000.

Collector Angle

There is a set of useful relationships among these angles. Equations relating the angle of incidence of beam radiation on a surface, θ, to the other angles are:

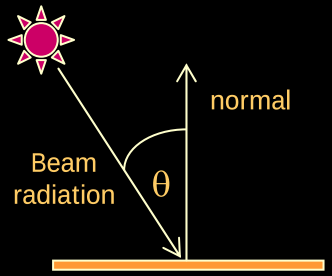


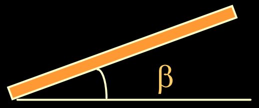
$$\delta = 23.45 \sin\left(360 \frac{284+n}{365}\right)$$

$$\begin{aligned}\cos\theta = {} & \sin\delta \sin\phi \cos\beta \\ & - \sin\delta \cos\phi \sin\beta \cos\gamma \\ & + \cos\delta \cos\phi \cos\beta \cos\omega \\ & + \cos\delta \sin\phi \sin\beta \cos\gamma \cos\omega \\ & + \cos\delta \sin\beta \sin\gamma \sin\omega\end{aligned}$$

and

$$\cos\theta = \cos\theta_z \cos\beta + \sin\theta_z \sin\beta \cos(\gamma_s - \gamma)$$

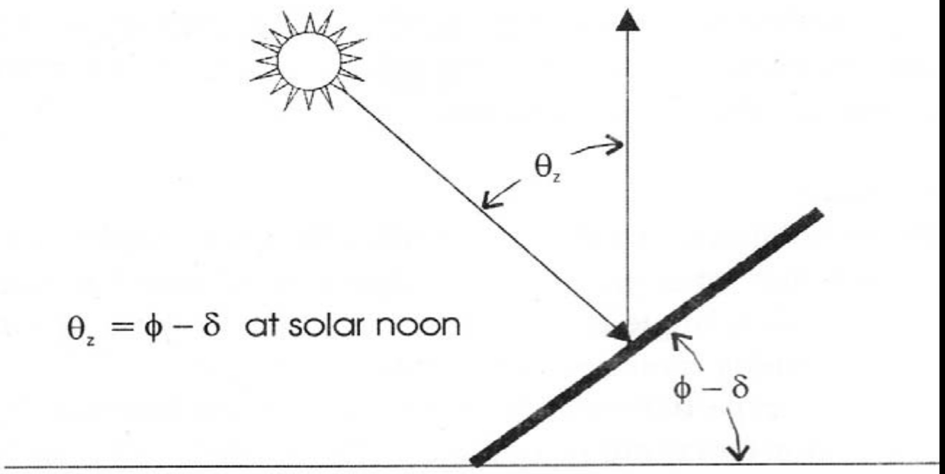




For horizontal surfaces. The angle of incidence is the zenith angle of the sun θ_z (0º or 90º when the sun is above the horizon). For this situation β=0

$$\cos\theta_z = \cos\theta \cos\delta \cos\omega + \sin\theta \sin\delta$$

Mounting Angle of a Fixed Collector

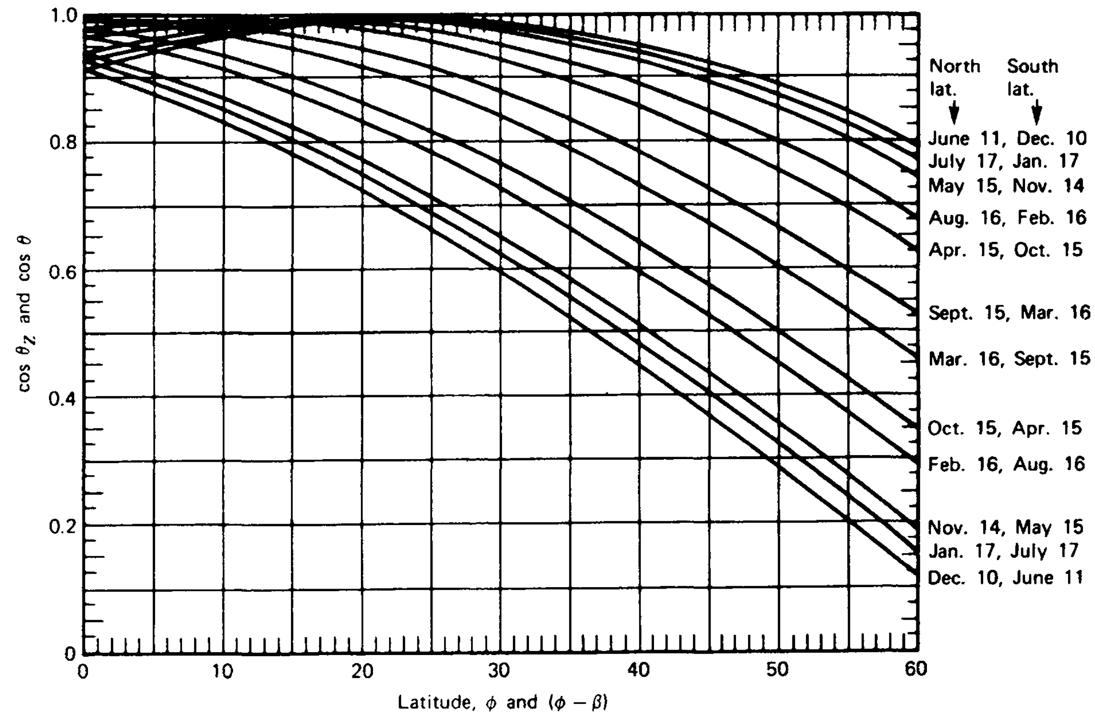


Optimizing the mounting angle of a fixed collector.

Source: Photovoltaic systems Engineering by Messenger & Ventre, CRC 2000.

Zenith Angle

Determination of Zenith angle (θ_z) for tilted surfaces: The zenith angle for tilted surfaces is determined from Whillier's curves given below.



Cos θ vs. ($\phi - \beta$) and cos θ_z vs. ϕ for hours 11 to 12 and 12 to 1 for surfaces tilted toward the equator. The columns on the right show dates for the curves for north and south latitudes. In south latitudes, use $|\phi|$. Adapted from Whillier (1975).

Tracking Surface Angles

For a plane rotated about a horizontal east-west axis with single daily adjustment of beam radiation being normal to the surface at noon each day:

$$\cos\theta = \sin^2\delta + \cos^2\delta\cos\omega$$

The slope of the surface is fixed for each day: $\beta = |\phi - \delta|$

If $(\phi - \delta) > 0$, $\gamma = 0^o$; $(\phi - \delta) < 0$, $\gamma = 180^o$

At solar noon $\phi_z = \phi - \delta$

Sunset Hour Angle

The *sunset hour angle* ω_s, when $\theta_z = 90°$ is given by

$$\cos\omega_s = -\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} = -\tan\phi\tan\delta$$

Number of daylight hours: If latitude and declination are known, the day length can be calculated from the formula:

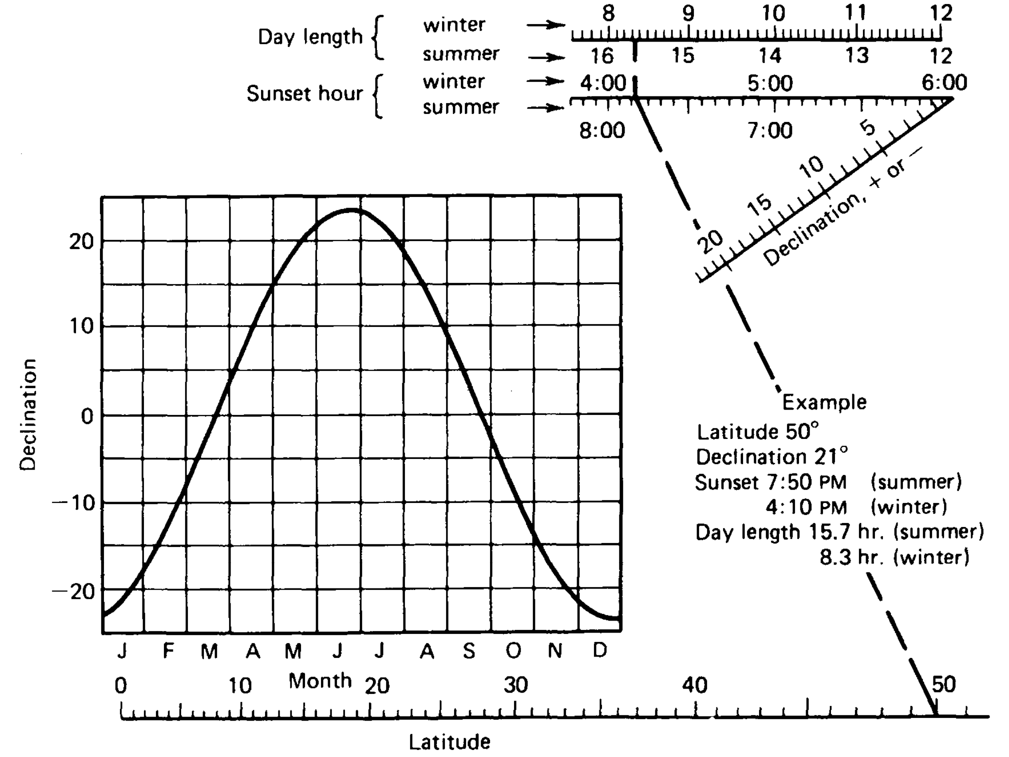
$$N = \frac{2}{15}\cos^{-1}\left(-\tan\phi\tan\delta\right)$$

$$\omega = \frac{12 - T}{24} \times 360^o$$

Where T is the time of day expressed with respect to solar midnight on a 24 hr clock.

Nomogram

Information on latitude (ϕ) and declination (δ) leads directly to times of sunset and day length for either hemisphere. These parameters can be conveniently determined from the nomogram (shown on the side) as devised by Whillier (1965b).



Nomogram to determine time of sunset and day length.

Sunshine Days

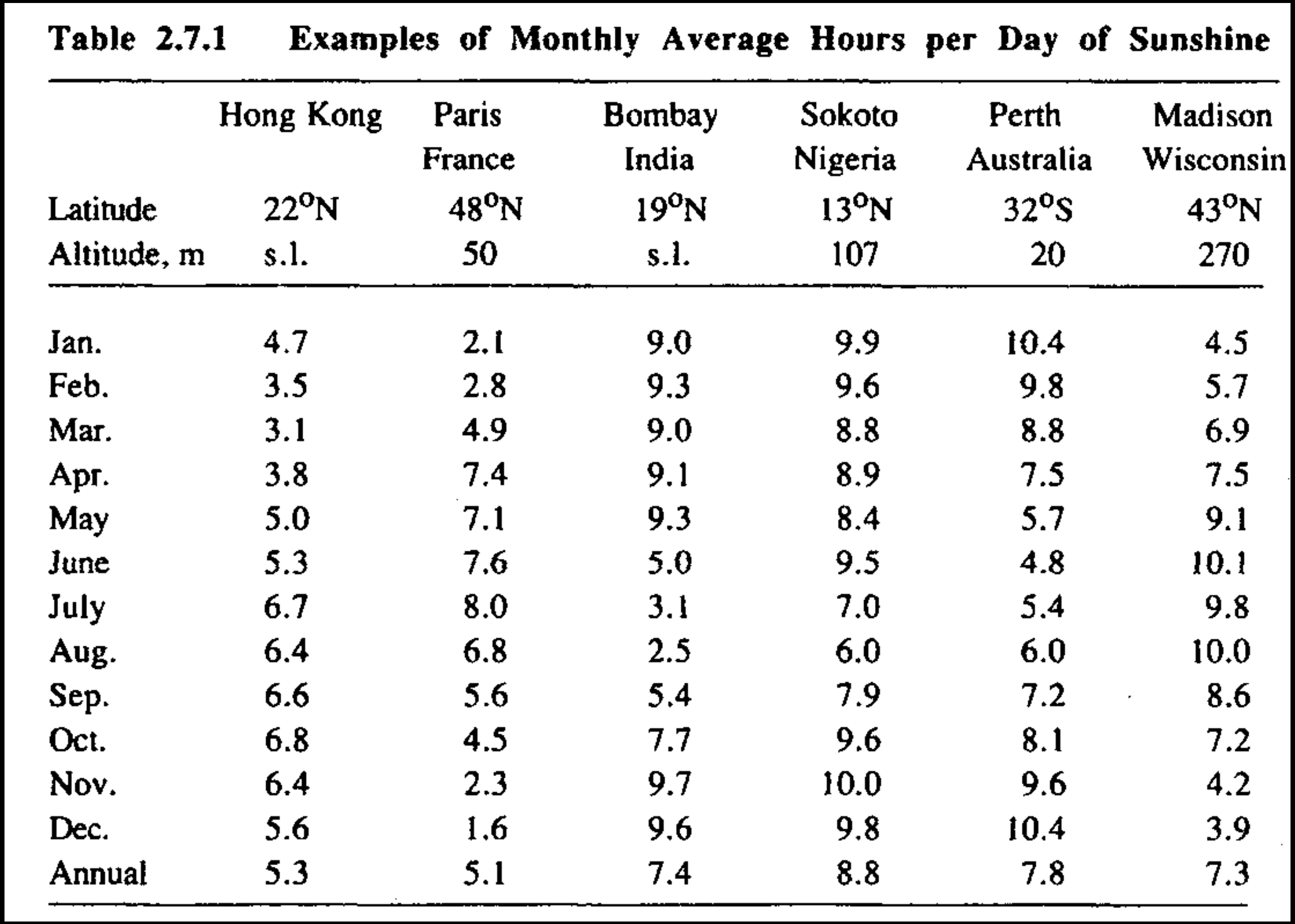


Table 2.7.1 Examples of Monthly Average Hours per Day of Sunshine

	Hong Kong	Paris France	Bombay India	Sokoto Nigeria	Perth Australia	Madison Wisconsin
Latitude	22°N	48°N	19°N	13°N	32°S	43°N
Altitude, m	s.l.	50	s.l.	107	20	270
Jan.	4.7	2.1	9.0	9.9	10.4	4.5
Feb.	3.5	2.8	9.3	9.6	9.8	5.7
Mar.	3.1	4.9	9.0	8.8	8.8	6.9
Apr.	3.8	7.4	9.1	8.9	7.5	7.5
May	5.0	7.1	9.3	8.4	5.7	9.1
June	5.3	7.6	5.0	9.5	4.8	10.1
July	6.7	8.0	3.1	7.0	5.4	9.8
Aug.	6.4	6.8	2.5	6.0	6.0	10.0
Sep.	6.6	5.6	5.4	7.9	7.2	8.6
Oct.	6.8	4.5	7.7	9.6	8.1	7.2
Nov.	6.4	2.3	9.7	10.0	9.6	4.2
Dec.	5.6	1.6	9.6	9.8	10.4	3.9
Annual	5.3	5.1	7.4	8.8	7.8	7.3

Radiation on Horizontal Surface

Extraterrestrial radiation on a horizontal surface: This is defined as the theoretically possible radiation (G_o) that would be available if there were no atmosphere.

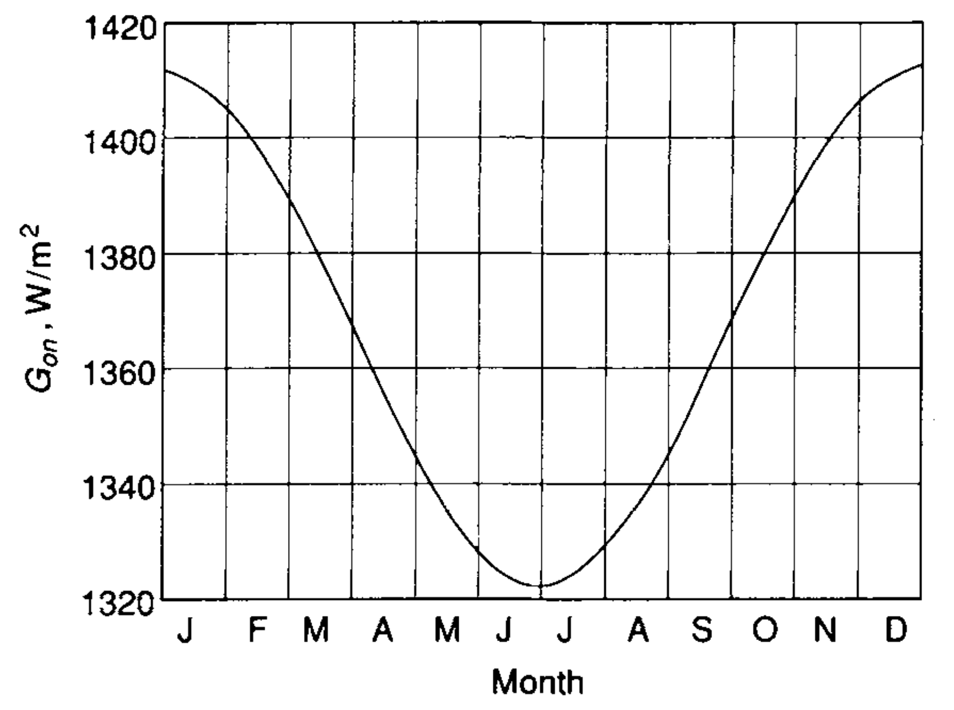
$$G_o = G_{sc}\left(1 + 0.033\cos\frac{360n}{365}\right)\left(\cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta\right)$$

G_{SC} is the solar constant and *n* is the day of the year.

Variation of Extraterrestrial Radiation

$$G_{on} = G_{sc}\left(1 + 0.033\cos\frac{360n}{365}\right)$$

Where G_{on} is the radiation measured on the plane normal to the radiation on the n^{th} day of the year.



Radiation on Horizontal Surface

The integrated daily (sunrise to sunset) extraterrestrial radiation (H_o) on a horizontal surface is given by:

$$H_o = \frac{24\,x\,3600 G_{SC}}{\pi}\left(1 + 0.033\cos\frac{360\,n}{365}\right)\left(\cos\phi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\phi\sin\delta\right)$$

where G_{SC} is in watts/m^2

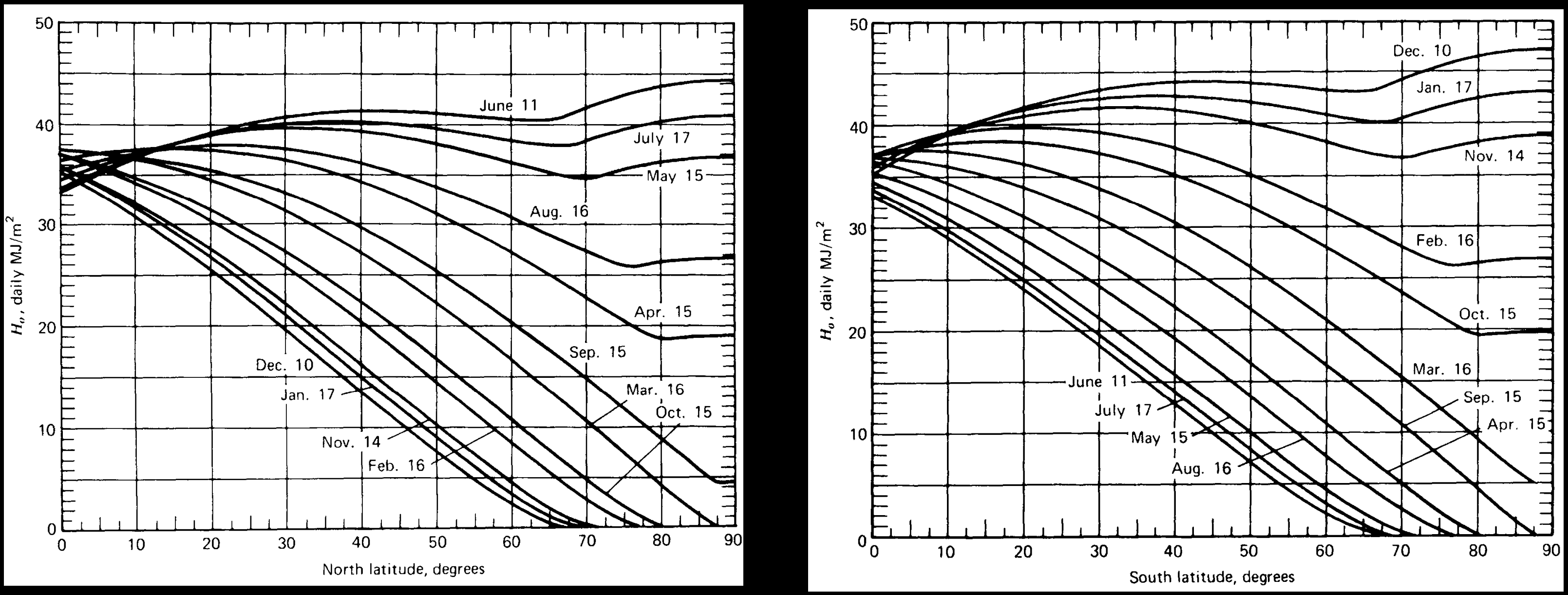
ω_S, the sunset hour angle, is in degrees

H_o is in joules/m^2

Radiation on Horizontal Surface

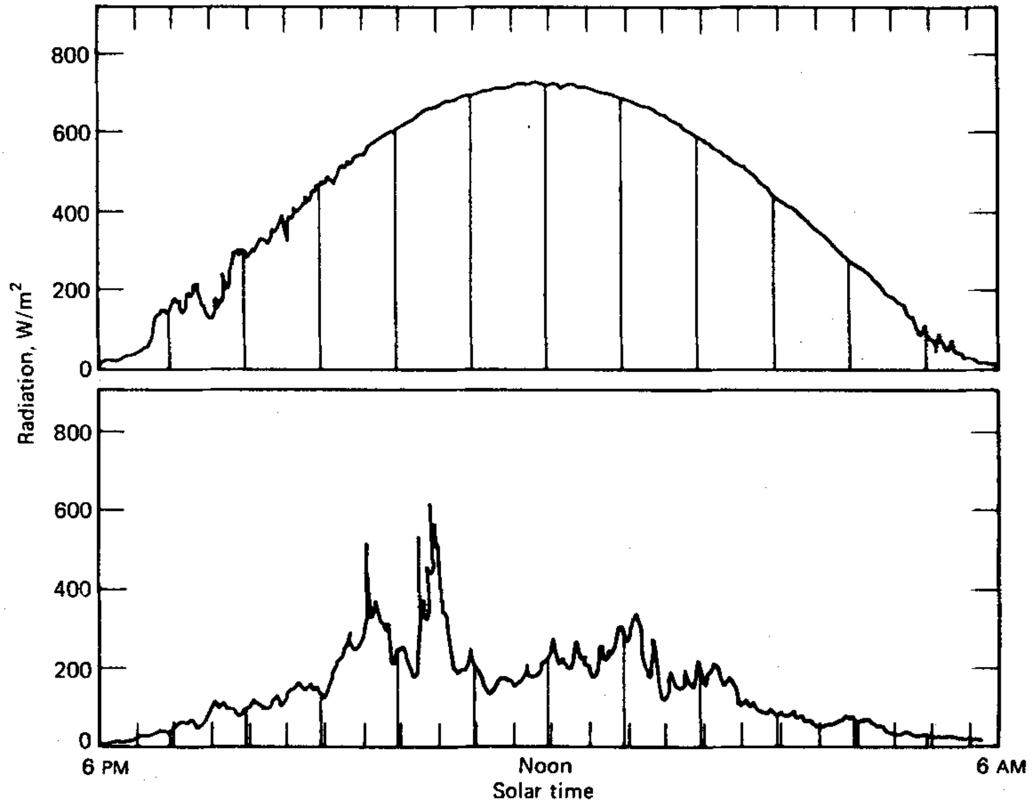
H_o as a function of latitude for the northern and southern hemispheres:

- Curves are for dates that give the mean radiation for the month
- Values of H_o for any day can be estimated by interpolation.



Extraterrestrial daily radiation on a horizontal surface. The curves are for the mean days of the month from Table 1.6.1.

Solar Radiation Data on Horizontal Surface



Total (beam and diffuse) solar radiation on a horizontal surface vs. time for clear and largely cloudy day, latitude 43°, for days near the equinox.

Solar Irradiance

In the design of photovoltaic systems the irradiation over one day is of particular significance. Solar irradiance integrated over a period of time is called solar irradiance (total power from a radiant source falling on a unit area).

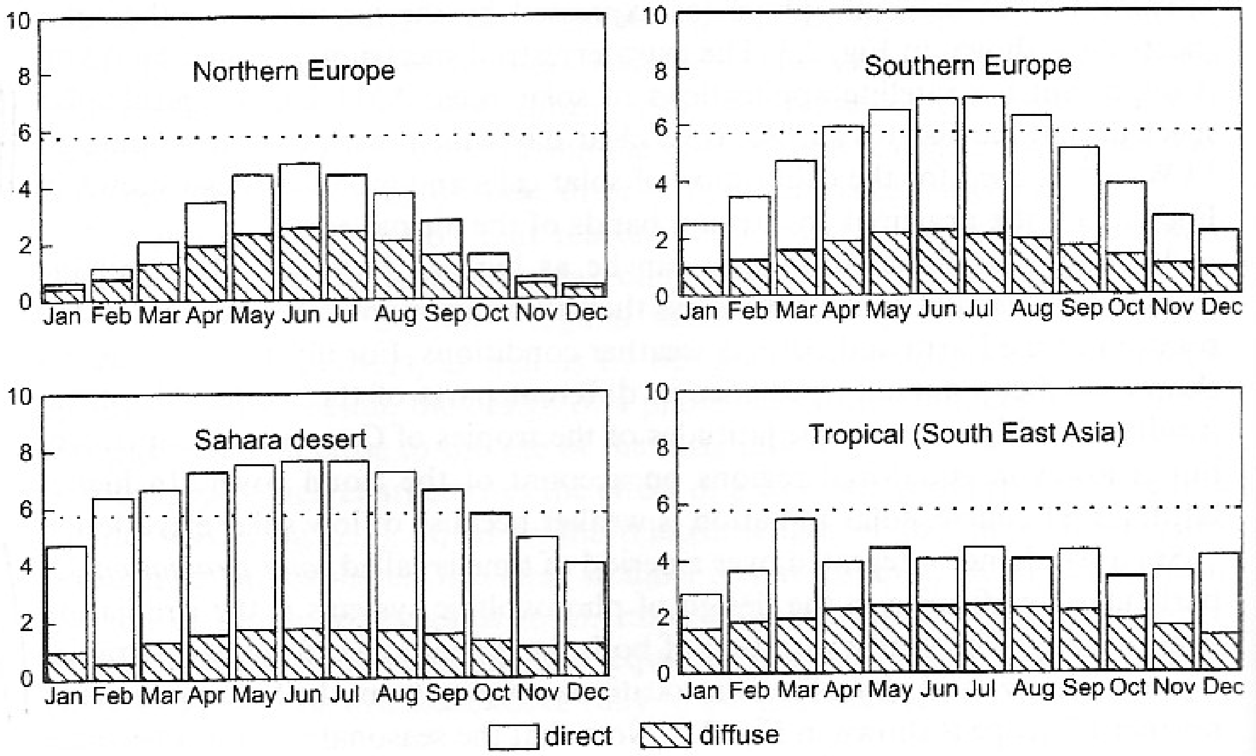
The average daily solar radiation on the ground G_{av} (averaged both over the location and time of the year) can be obtained using the following relationship.

$$G_{av} = 0.7 \times 342 \times 24h = 5.75 kWh / day$$

Where the average irradiance outside the atmosphere = 342 W/m^2

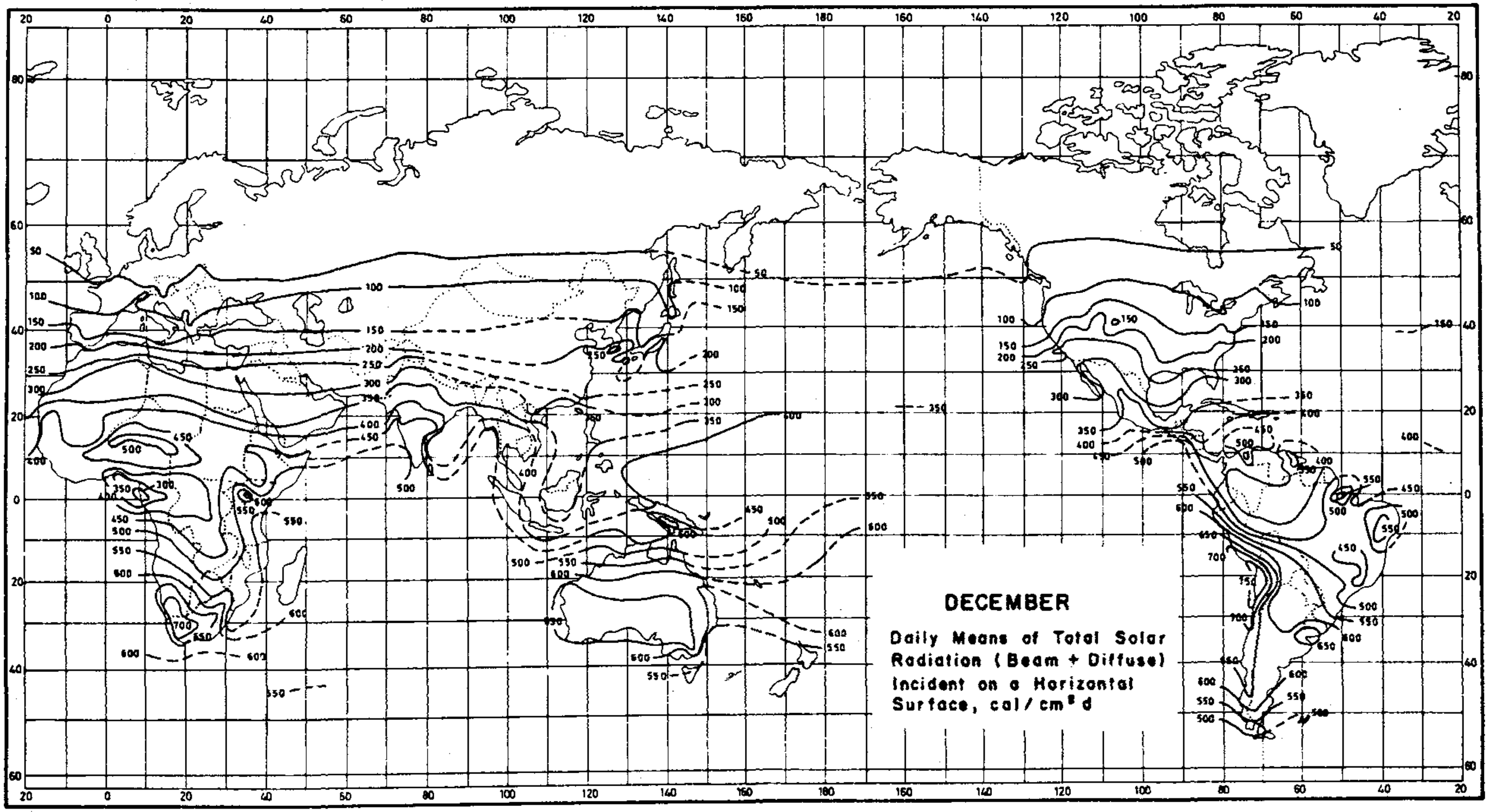
Solar radiation observed on the earth’s surface is 30% lower on account of scattering and reflection of radiation.

Mean Daily Solar Radiation



The yearly profile of mean daily solar radiation (in kWh/m^2) as a function of geographic location. Dashed line indicates the world average

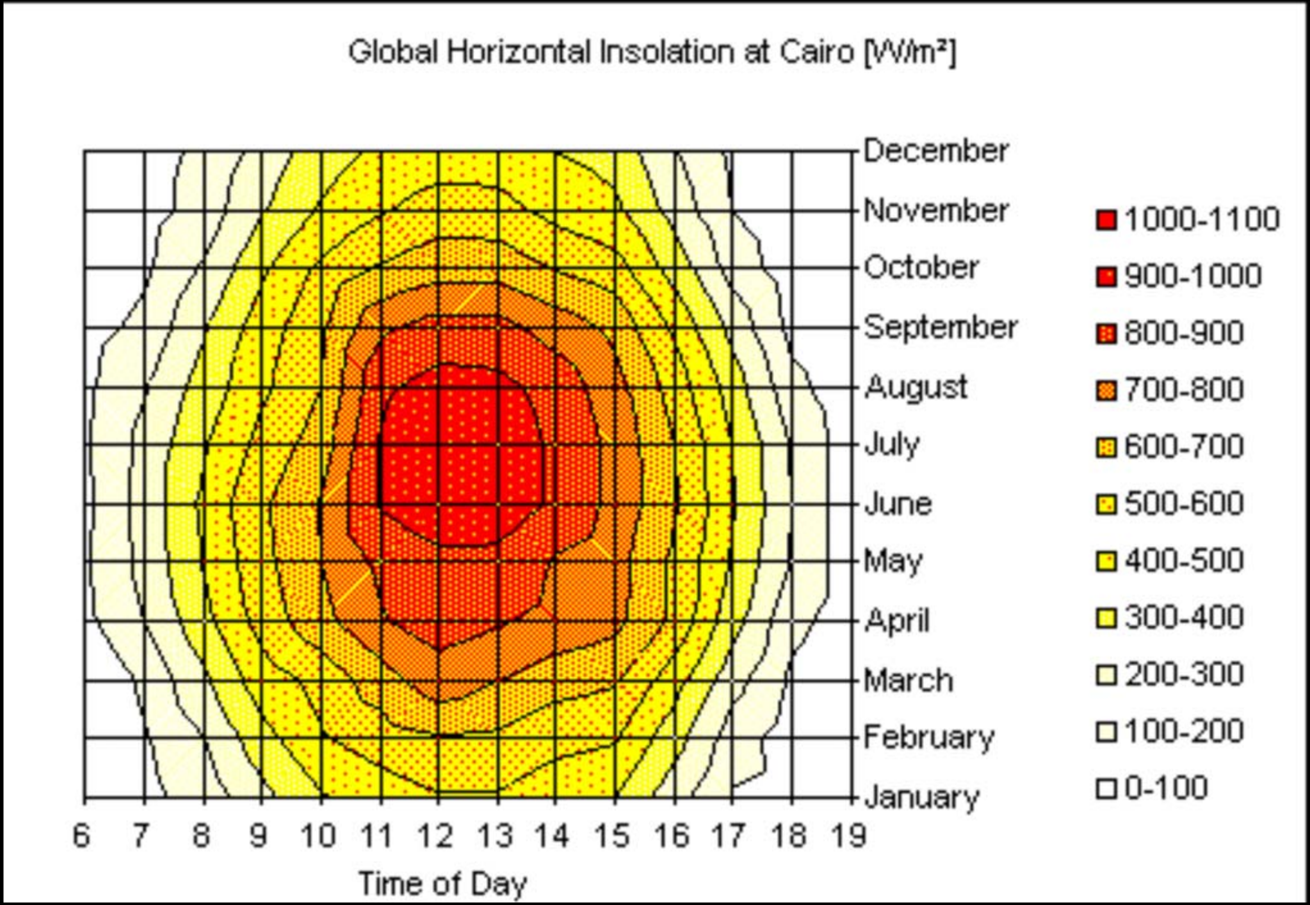
Solar Radiation Data on Horizontal Surface



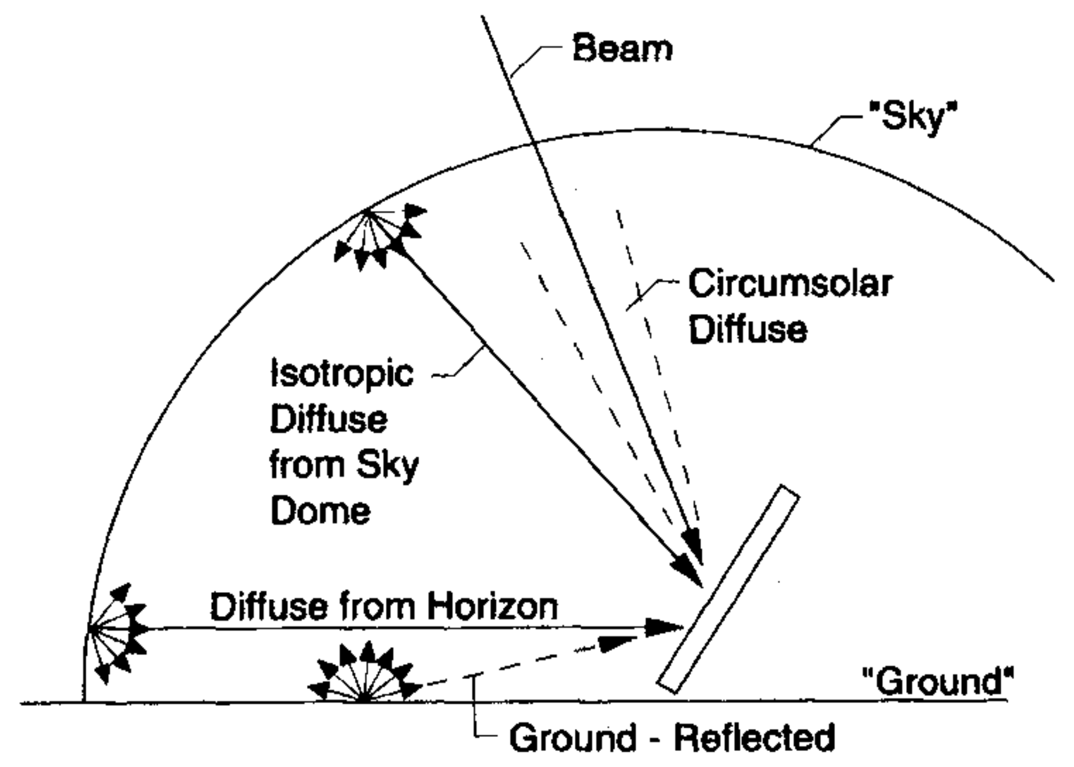
Average daily radiation on horizontal surfaces for December. Data are in cal/cm^2, the traditional units. Adapted from deJong (1973) and Löf et al. (1966a).

1 cal/cm^2 = 41.87 kJ/m^2; 1 kWh = 3.6 MJ

Global Horizontal Insolation



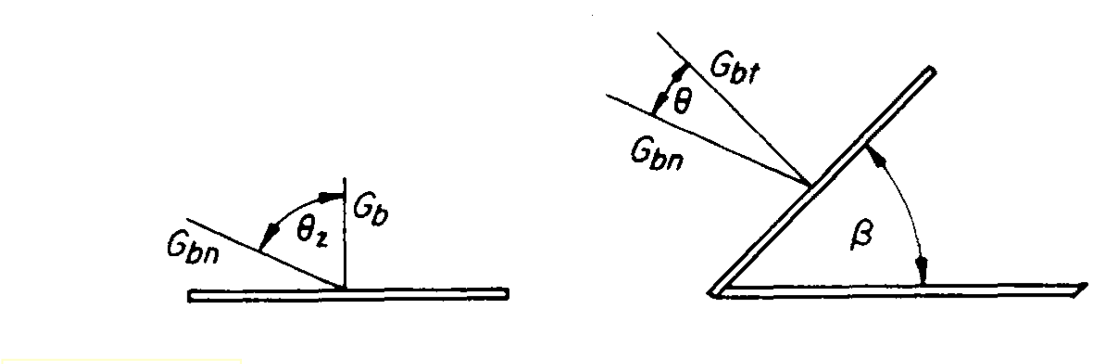
Solar Radiation in the Atmosphere



Tilted Surfaces

Ratio of Beam radiation on tilted surface to that on horizontal surface (R_b) : The most commonly available data are total radiation for hours and or days whereas the need is for beam and diffuse radiation on the plane of a collector (normally tilted).

$$R_b = \frac{G_{b,T}}{G_b} = \frac{G_{b,n}\cos\theta}{G_{b,n}\cos\theta_z} = \frac{\cos\theta}{\cos\theta_z}$$



Beam radiation on horizontal and tilted surfaces.

For horizontal surfaces: $\cos\theta_z = \cos\theta\cos\delta\cos\omega + \sin\theta\sin\delta$

Zenith Angle Effect

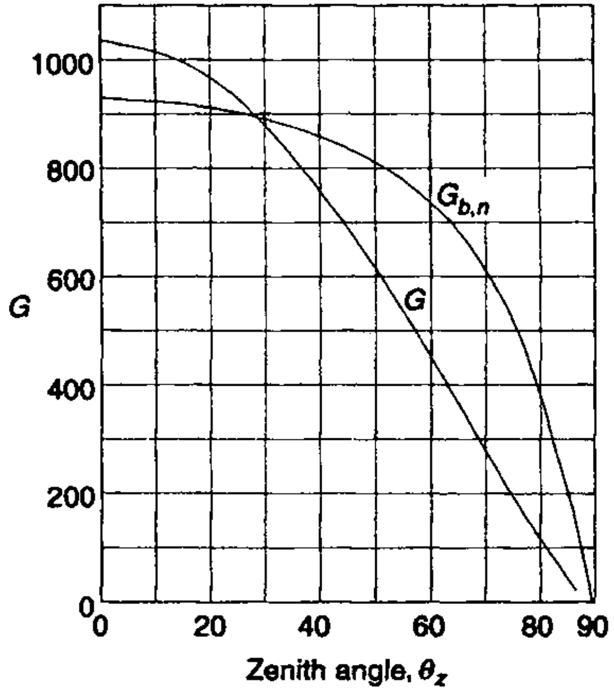


Figure 2.8.1 Total horizontal radiation and beam normal radiation for the ASHRAE standard atmosphere. Data are from Farber and Morrison (1977).

Tracking Surfaces

Angles for Tracking Surfaces: Some solar collectors “track” the sun by moving in prescribed ways to minimize the angle of incidence of beam radiation on their surfaces and thus maximize the incident beam radiation.

Classification of tracking systems by motion:

- Rotation about a single axis
- Rotation about two axes

The axis described above could have any orientation but is usually horizontal east-west, horizontal north-south, vertical or parallel to the earth’s axis.

For relationships between the angles of incidence and the surface azimuth angles, refer to the handout.

Inclination Effect

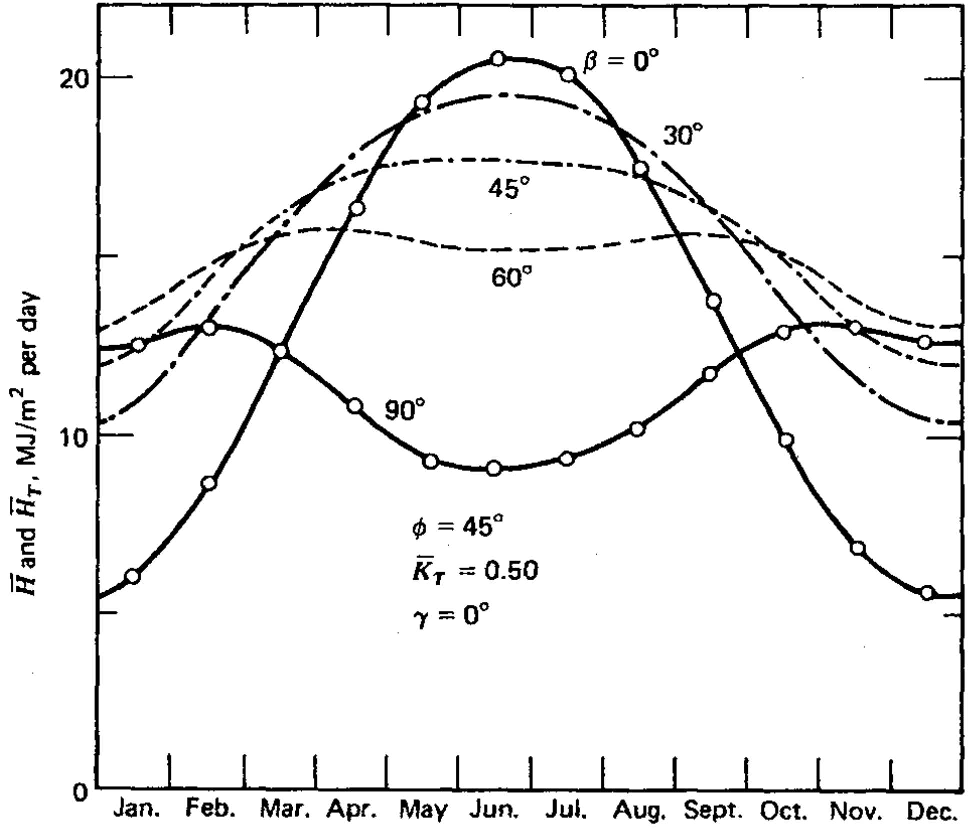


Figure 2.21.1 Variation in estimated average daily radiation on surfaces of various slopes as a function of time of year for a latitude of 45°, $\bar{K}_T$ of 0.50, surface azimuth angle of 0°, and a ground reflectance of 0.20.

Solar radiation on an inclined surface

Step 1: Obtain the site data to determine the separate diffusive and beam contributions to the global irradiation (G) on the horizontal plane.

Use the extraterrestrial daily irradiation B_o, as a reference and calculate the *clearness index* K_T. This describes the average attenuation of solar radiation by the atmosphere at a given site during a given month.

$$K_T = \frac{G}{B_o}$$

Note: In the evaluation of B_o, the variation of the extraterrestrial irradiance on account of the eccentricity of the earth's orbit (~ ± 3%) is taken into account

$$B_o = \frac{24}{\pi} S \left\{ 1 + 0.33 \cos\left(\frac{2\pi d_n}{365} \right) \right\} \left(\cos\varphi \cos\delta \sin\omega_s + \omega_s \sin\varphi \sin\delta \right)$$

S: Solar constant = 1367 W/m^2 ; d_n *; number of the day in the year;* Latitude(ϕ),

Declination(δ), *Sunrise hour angle* $\omega_s = \cos^{-1}(-\tan\phi \tan\delta)$

Source: Solar Electricity by Tomas Markvart, Wiley, 2003

Solar radiation on an inclined surface

Step 2: The diffuse radiation is obtained using the empirical rule that the diffuse fraction is a universal function of clearness index.

$$\frac{D}{G} = f\left(K_T\right) = 1 - 1.13K_T$$

$$B = G - D$$

B: Beam irradiation

D: Diffuse irradiation

G: Global daily irradiation

Solar radiation on an inclined surface

Step 3: Appropriate angular dependences of each component are used to determine the diffuse and beam irradiation on the inclined surface. With the allowance for reflectivity of the surrounding area. The total daily irradiation on the inclined surface is obtained by adding the three contributions.

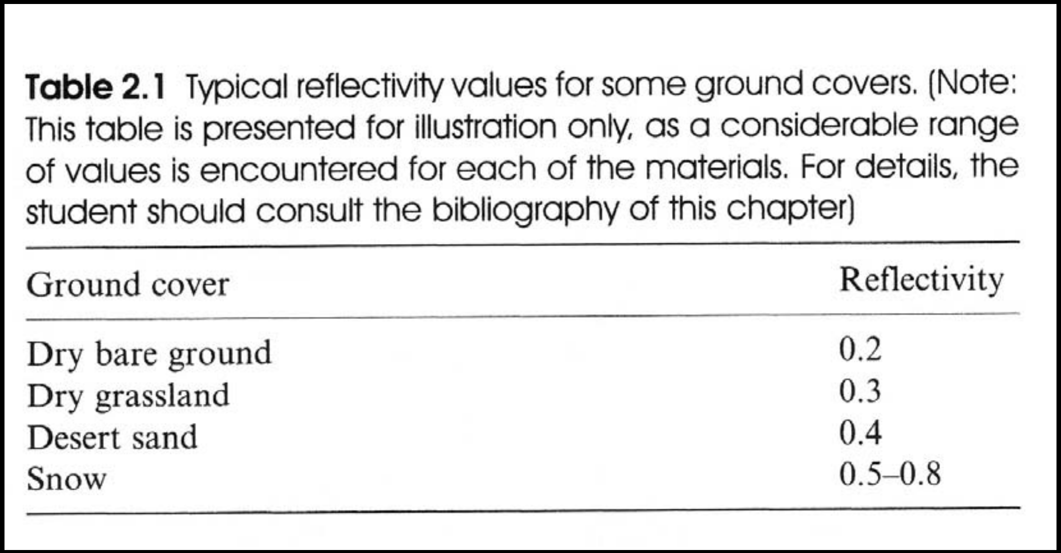


Table 2.1 Typical reflectivity values for some ground covers. (Note: This table is presented for illustration only, as a considerable range of values is encountered for each of the materials. For details, the student should consult the bibliography of this chapter)

Ground cover	Reflectivity
Dry bare ground	0.2
Dry grassland	0.3
Desert sand	0.4
Snow	0.5–0.8

Solar radiation on an inclined surface

Beam radiation:

The beam radiation B(β) on a south-facing panel inclined at an angle β to the horizontal surface is given by

$$B(\beta) = B\frac{\cos(\phi-\beta)\cos\delta\sin\omega_o + \omega_o\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \omega_s\sin\phi\sin\delta}$$

$$\omega_o = \min\{\omega_s, \omega_s^{'}\}$$

$$\omega_s^{'} = \cos^{-1}\{-\tan(\phi-\beta)\tan\delta\}$$

Where ω_s and ω_s' are the sunrise angle above the horizon and the sunrise angle above a plane inclined at an angle β to the horizontal

Solar radiation on an inclined surface

Diffusive radiation:

Assuming it is distributed isotropically over the sky dome, the diffusive radiation on the inclined surface is given by

$$D(\beta) = \frac{1}{2}(1 + \cos\beta)D$$

Albedo (ground reflected) radiation:

It is generally small and a simple isotropic model is sufficient as given below.

$$R(\beta) = \frac{1}{2}(1 + \cos\beta)\rho G$$

ρ Is the reflectivity of the ground.

Solar radiation on an inclined surface

Total Global irradiation:

$$G(\beta) = B(\beta) + D(\beta) + R(\beta)$$

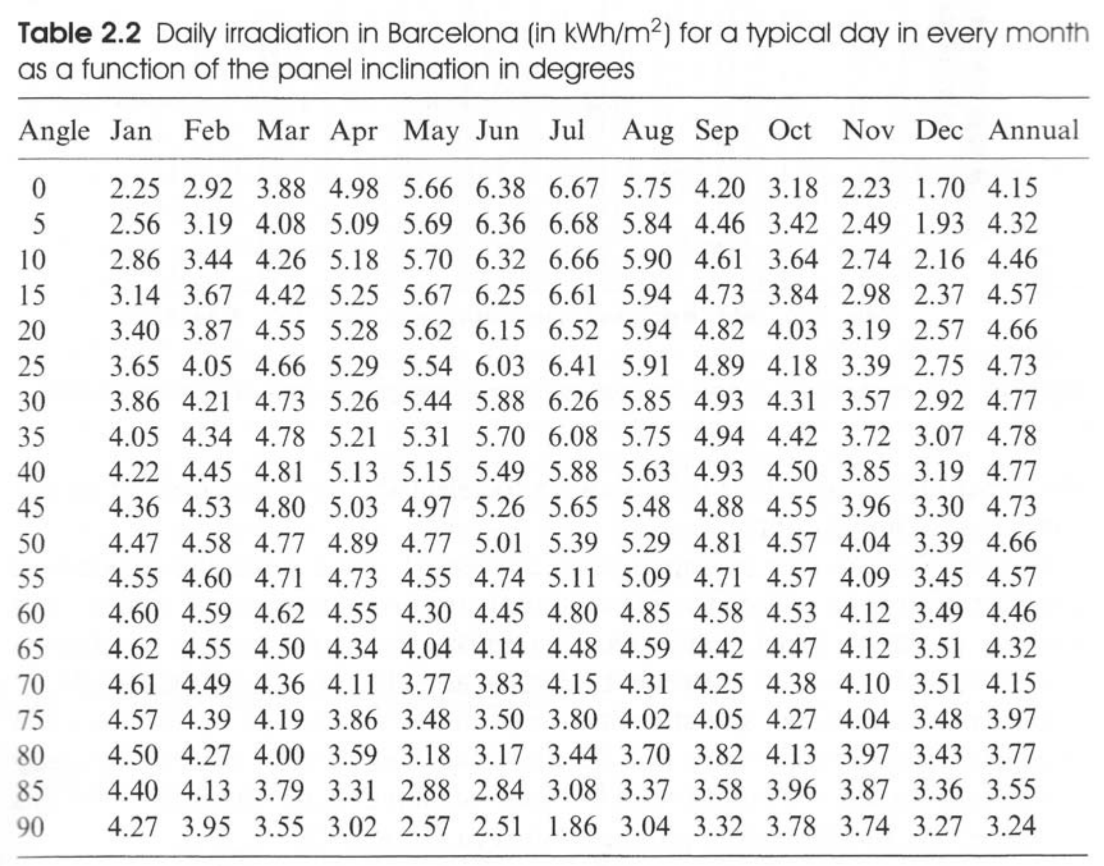


Table 2.2 Daily irradiation in Barcelona (in kWh/m^2) for a typical day in every month as a function of the panel inclination in degrees

Angle	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
0	2.25	2.92	3.88	4.98	5.66	6.38	6.67	5.75	4.20	3.18	2.23	1.70	4.15
5	2.56	3.19	4.08	5.09	5.69	6.36	6.68	5.84	4.46	3.42	2.49	1.93	4.32
10	2.86	3.44	4.26	5.18	5.70	6.32	6.66	5.90	4.61	3.64	2.74	2.16	4.46
15	3.14	3.67	4.42	5.25	5.67	6.25	6.61	5.94	4.73	3.84	2.98	2.37	4.57
20	3.40	3.87	4.55	5.28	5.62	6.15	6.52	5.94	4.82	4.03	3.19	2.57	4.66
25	3.65	4.05	4.66	5.29	5.54	6.03	6.41	5.91	4.89	4.18	3.39	2.75	4.73
30	3.86	4.21	4.73	5.26	5.44	5.88	6.26	5.85	4.93	4.31	3.57	2.92	4.77
35	4.05	4.34	4.78	5.21	5.31	5.70	6.08	5.75	4.94	4.42	3.72	3.07	4.78
40	4.22	4.45	4.81	5.13	5.15	5.49	5.88	5.63	4.93	4.50	3.85	3.19	4.77
45	4.36	4.53	4.80	5.03	4.97	5.26	5.65	5.48	4.88	4.55	3.96	3.30	4.73
50	4.47	4.58	4.77	4.89	4.77	5.01	5.39	5.29	4.81	4.57	4.04	3.39	4.66
55	4.55	4.60	4.71	4.73	4.55	4.74	5.11	5.09	4.71	4.57	4.09	3.45	4.57
60	4.60	4.59	4.62	4.55	4.30	4.45	4.80	4.85	4.58	4.53	4.12	3.49	4.46
65	4.62	4.55	4.50	4.34	4.04	4.14	4.48	4.59	4.42	4.47	4.12	3.51	4.32
70	4.61	4.49	4.36	4.11	3.77	3.83	4.15	4.31	4.25	4.38	4.10	3.51	4.15
75	4.57	4.39	4.19	3.86	3.48	3.50	3.80	4.02	4.05	4.27	4.04	3.48	3.97
80	4.50	4.27	4.00	3.59	3.18	3.17	3.44	3.70	3.82	4.13	3.97	3.43	3.77
85	4.40	4.13	3.79	3.31	2.88	2.84	3.08	3.37	3.58	3.96	3.87	3.36	3.55
90	4.27	3.95	3.55	3.02	2.57	2.51	1.86	3.04	3.32	3.78	3.74	3.27	3.24

Solar radiation on an inclined surface

Daily irradiation in Barcelona

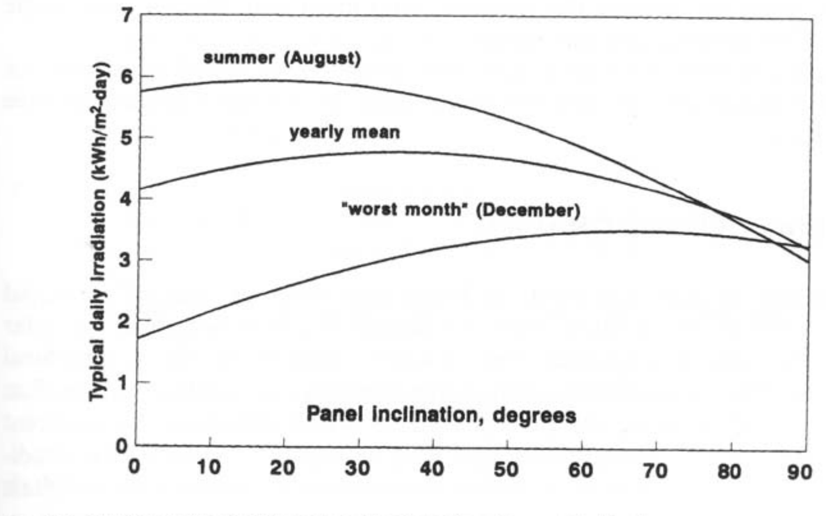


Fig. 2.9 Daily irradiation in Barcelona as a function of panel inclination

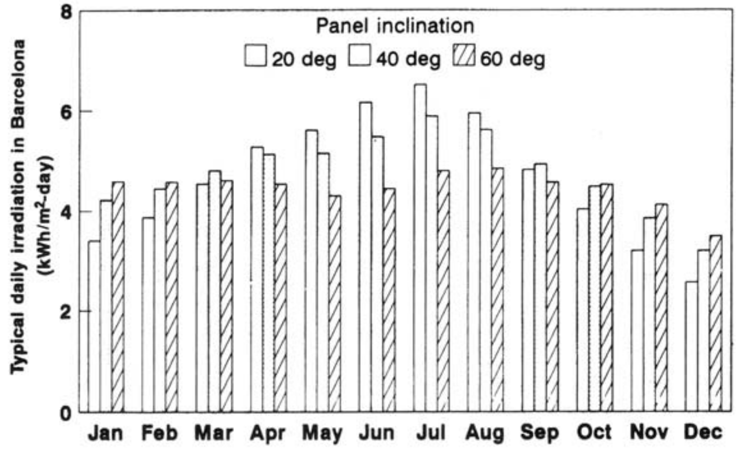


Fig. 2.10 Daily irradiation in Barcelona over the year for selected angles of panel inclination

Solar radiation on an inclined surface

South Facing

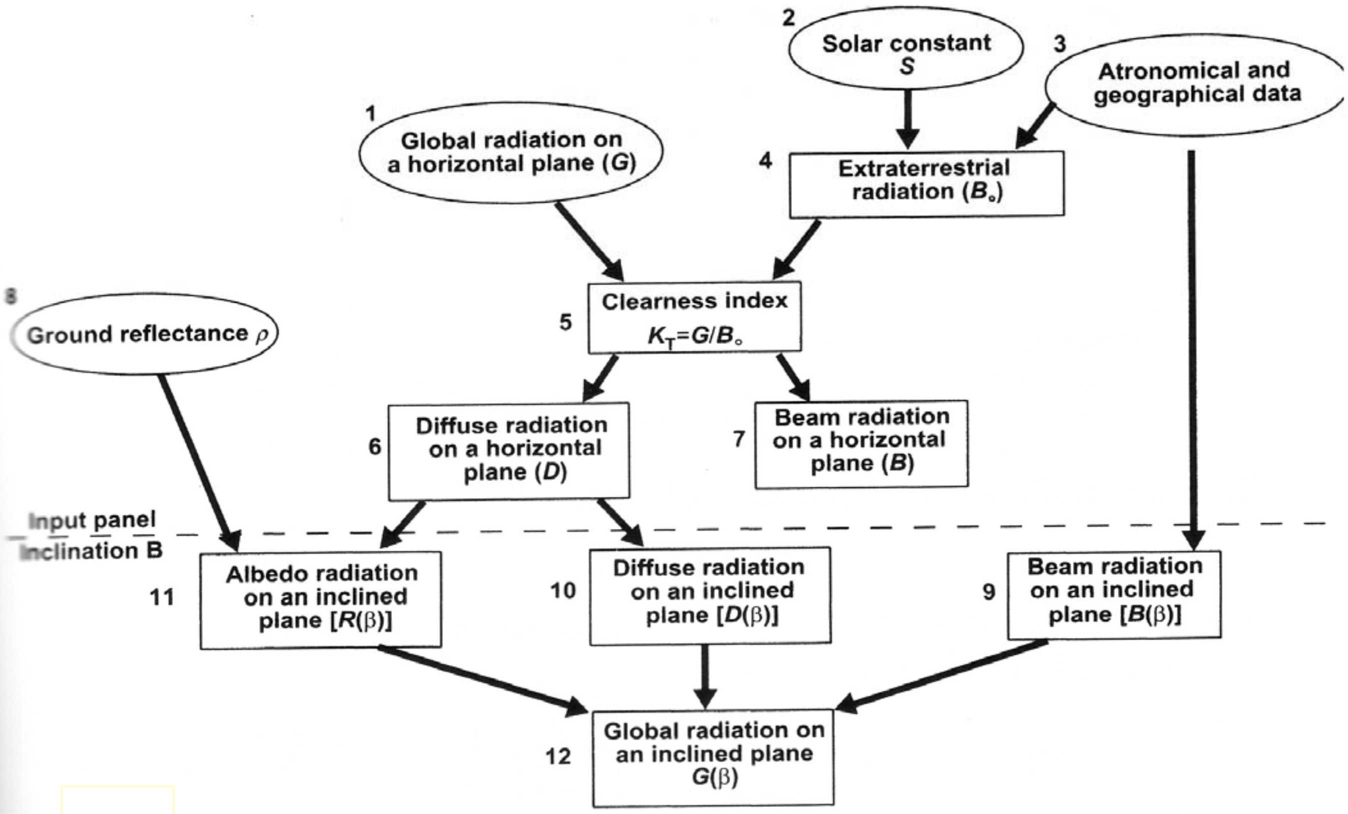


Fig. 2.8 Calculation of global radiation on an inclined plane. Oval boxes represent input data, and rectangular boxes indicate steps in the calculations

Shading Effects

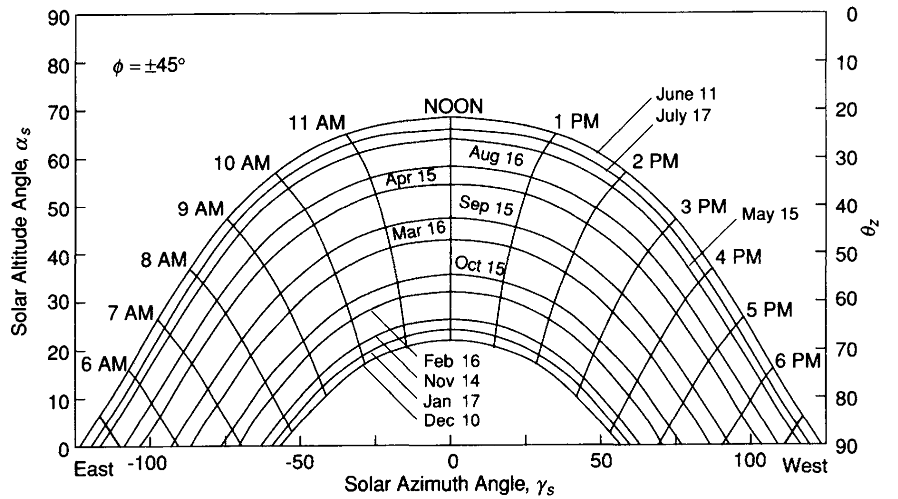
Problems associated with Shading:

- Collectors or other receivers by near-by trees, buildings etc.
- Shading of collectors in subsequent rows other than the first row in a multi-row array.
- Shading of windows by overhangs and wingwalls.

Shading Effects

Estimating the amount of time the sun is obscured by an obstruction:

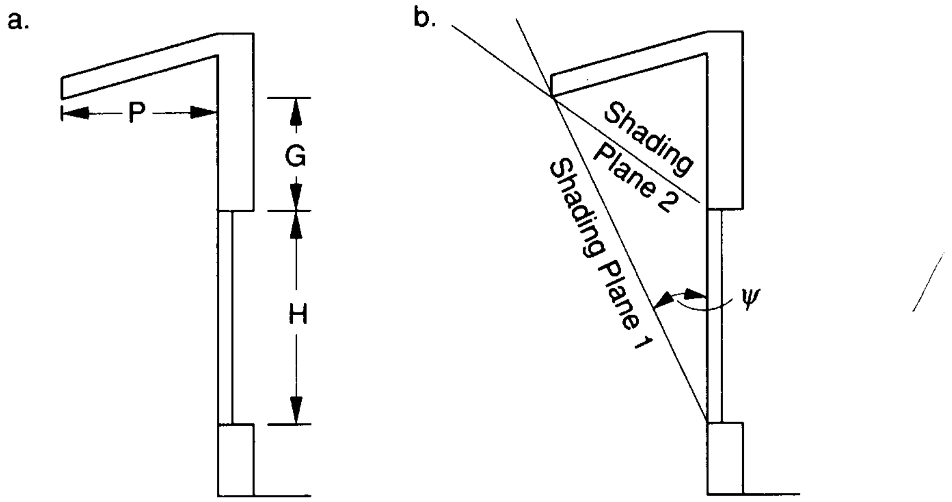
- Calculate the object azimuth angle (γ_o) and the object altitude angle (α_o) . Note that the "object" is the cause of shading.
- Locate these angular co-ordinates on the Solar plot and join by a smooth curve.
- Estimate the amount of time the Sun is obscured.



Solar position plot for $\pm 45^{o}$ latitude. Solar altitude angle and solar azimuth angle are functions of declination and hour angle, indicated on the plots by dates and times. The dates shown are for northern hemisphere; for southern hemisphere use the corresponding dates as indicated in Figure 1.8.2. See Appendix H for other latitudes.

Shading Effects

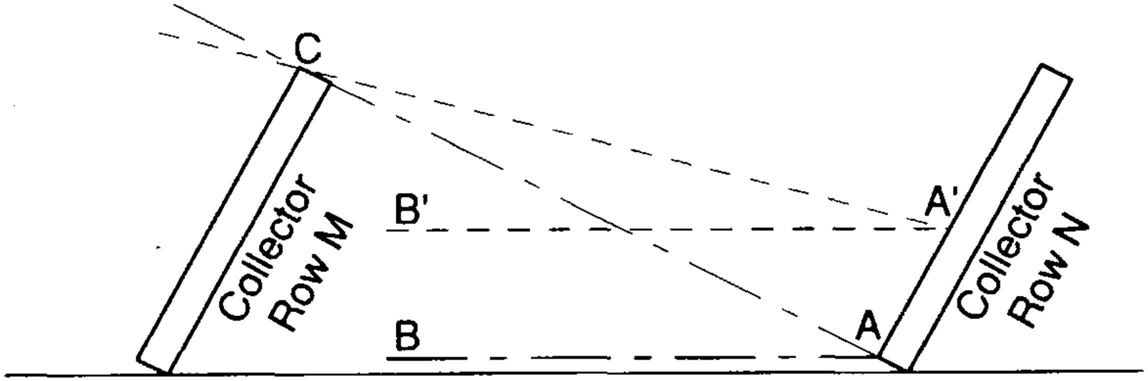
Shading due to overhangs and wingwalls: Architectural features like overhangs and wingwalls shade windows from beam radiation. The solar position charts can be used to determine when points on the receiver are shaded.



(a) Cross section of a long overhang showing projection, gap, and height. (b) Section showing shading planes.

Shading Effects

Shading due to multi-row array of collectors: In a multi-row collector array, the first row is unobstructed, but subsequent rows may be partially shaded as shown.



Section of two rows of a multirow collector array.

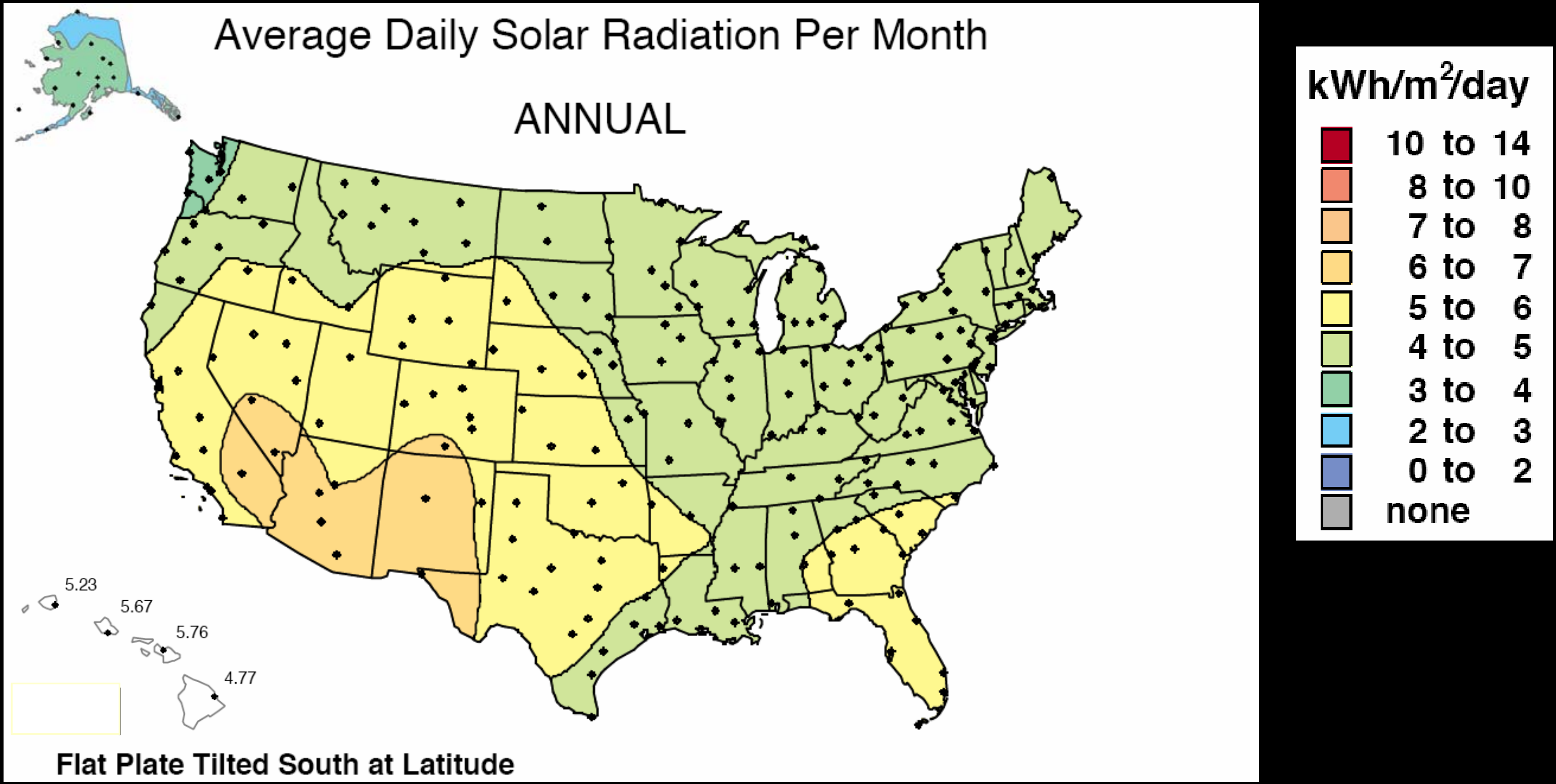
Shading Effects

Effect of Profile angle on shading due to multi-row array of collectors: The "Profile angle" is the angle through which a plane that is initially horizontal must be rotated about an axis in the plane of the surface in question in order to include the sun.

When the collectors are long in extent so the end effects are negligible, the profile angle provides a useful means of determining shading.

Referring to the previous slide, if the profile angle is greater than angle CAB, no point on row N will be shaded by row M. On the contrary, if the profile angle is less than angle CAB, the portion of row N below point A' will be shaded from beam radiation.

US Radiation Map



This map shows the general trends in the amount of solar radiation received in the United States and its territories. It is a spatial interpolation of solar radiation values derived from the 1961-1990 National Solar Radiation Data Base (NSRDB). The dots on the map represent the 239 sites of the NSRDB.

Maps of average values are produced by averaging all 30 years of data for each site. Maps of maximum and minimum values are composites of specific months and years for which each site achieved its maximum or minimum amounts of solar radiation.

Though useful for identifying general trends, this map should be used with caution for site-specific resource evaluations because variations in solar radiation not reflected in the maps can exist, introducing uncertainty into resource estimates.

Maps are not drawn to scale.

Flat-plate collector facing south at fixed tilt equal to the latitude of the site: Capturing the maximum amount of solar radiation throughout the year can be achieved using a tilt angle approximately equal to the site's latitude.

Source: NREL

Homework

1. Estimate the monthly average electricity usage for a typical 2500 sq. ft home in Tallahassee consisting of four bedrooms and two bath rooms. All the appliances are electric including the hot water heater. State your assumptions clearly.

2. Calculate the daily irradiation in Tallahassee in kWh/m^2 for every month using the optimum solar flat plate collector inclination. Determine the optimum inclination to maximize the annual solar radiation collection. Calculate monthly average hours of sunshine and the total annual solar radiation collected in terms of kWh/m^2.

Tallahassee: Latitude: 30.47N and Longitude: -84.28W

Source: Solar Engineering of Thermal Processes by Duffie & Beckman, John Wiley & Sons, 1991